# FUZZY MATHEMATICS

A project submitted to the University of Kerala.
*In Partial Fulfillment for the Award of the Degree of*

**BACHELOR OF SCIENCE**
**IN**
**MATHEMATICS**

By



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COURSE CODE: MM 1646
MARCH 2024

# DECLARATION

I hereby declare that the project work entitled "FUZZY MATHEMATICS" submitted to Kerala University is a record of an original work done by us under the guidance of Dr Oleena SH, Assistant Professor,Department of Mathematics,All Saint's college,Trivandrum and this project work is submitted in the partial fulfillment of the requirements for the award of the degree of Bachelor in Mathematic. The results embodied in this project have not been submitted to any other University or Institute for the award of any degree or diploma.

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# CERTIFICATE

This is to certify that the Project entitled "FUZZY MATHEMATICS" is the bonafide work carried out by

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under my supervision and guidance and it has not been submitted earlier or elsewhere for similar purposes according to the best of my knowledge and belief.

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# ACKNOWLEDGEMENT

I wish to express my deep sense of gratitude to Dr Oleena SH,Assistant Professor, Department of Mathematics, All Saint's college, Trivandrum, under whose supervision this project work has been carried out.I am indebted to her for this encouragement, guidance and constructive criticism throughout during the course of this work. I also wish to express my profound thanks to all other teachers for their heartly helpfulness. I must also thank my librarian and other non-teaching staff who helped me in one way or other,in fulfilling this work. I also express my sincere thanks to my classmates for their valuable suggestions.

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# INTRODUCTION

In mathematics there is no room for vagueness, for randomness, and for extremely small quantities. By introducing one of these qualities into mathematics, one can create alternative mathematics . But how do we introduce vagueness into mathematics? One very simple way to achieve this is to allow notions like "small," "large," and "few." However, another way is to modify the most basic object of mathematics, that is, to modify sets. In this respect, fuzzy mathematics is a form of alternative mathematics since it is based on a generalization of set membership.

In fuzzy mathematics, an element may belong to a degree to a set, while in ordinary mathematics, it either belongs or does not belong to a set. This simple idea has been applied to most fields of mathematics.It enables one to work in uncertain and ambiguous situations and solve ill-posed problems or problems with incomplete information.

This dissertation consists of four chapters.In the first chapter we discuss some preliminaries and definitions.In the second chapter, we discuss about Fuzzy arithmetics.In the third chapter,we discuss about fuzzy logics and operations and in the fourth chapter, some applications of fuzzy mathematics

# Chapter 1

# Preliminary

**Definition 1.1** A set is defined as a collection of distinct, well-defined objects forming a group.

**Definition 1.2** Let A and B be two sets. The **Cartesian product** of these sets is denoted by $A \times B$ and consists of all ordered pairs (a, b) with $a \epsilon A$ and $b \epsilon B$

**Definition 1.3** A **binary operation** * on a set A is a function mapping $A \times A$ to A defined as

$$(a, b) = a * b$$

**Definition 1.4** A function $f$ from A to B is defined as a rule that assigns each element of A.

**Definition 1.5** **Domain** of a function is the set of values for which the function is defined.

**Definition 1.6** **Range** of a function is the set of all function values.

**Definition 1.7** The **limit** of a function at a point a in its domain (if it exists) is the value that the function approaches as its argument approaches a.

**Definition 1.8** A function $f$ is **continuous** at some point c of its domain, if the limit of $f(x)$ as $x$ approaches to $c$ through the domain of $f$ exist and is equal to $f(c)$

$$i.e, \lim_{x \to c} f(x) = f(c)$$

**Definition 1.9** An **universal set** X is defined in the universe of discourse and it includes all possible elements related with the given problem. If we define a set A in the universal set X, we see the following relationships

$$A \subseteq X.$$

In this case, we say a set A is included the universal set X.

**Definition 1.10** If an element x is included in the set A, this element is called as a **member of the set** and the following notation is used.

$$x \epsilon A.$$

If the element x is not included in the set A, we use the following notation.

$$x \notin A.$$

**Definition 1.11** The **cardinality** of a set is a measure of a set's size,The cardinality of set A is denoted by $|A|$. If the cardinality —A— is a finite number, the set A is a finite set. If $|A|$ is infinite, A is an infinite set.

**Definition 1.12** The relative **complement set** of set A to set B consists of the elements which are in B but not in A. The complement set can be defined by the following formula.

$$B–A = \{x | x \epsilon B, x \epsilon A.\}$$

If the set B is the universal set X, then this kind of complement is an absolute complement set A . That is, $A = X - A$

**Definition 1.13** The complement of an empty set is the universal set, and vice versa.

$$\phi = XX = \phi$$

**Definition 1.14** The union of sets A and B is defined by the collection of whole elements of A and B.

$$A \cup B = x|x\epsilon A or x\epsilon B.$$

**Definition 1.15** The **intersection** $A \cap B$ consists of whose elements are commonly included in both sets A and B.

$$A \cap B = x|x\epsilon A and x\epsilon B.$$

**Definition 1.16** A decomposition of set A into disjoint subsets whose union builds the set A is referred to a partition. Suppose a partition of A is $\pi$,

$$\pi(A) = A_i|i\epsilon I, A_i \underline{c} A$$

then $A_i$ satisfies following three conditions.

1. $A_i \neq \phi$

2. $A_i \cap A_j = \phi, i \neq j, i, j\epsilon I$

3. $\bigcap_{i\epsilon I} A_i = A$

If there is no condition of (2), $\phi(A)$ becomes a cover or covering of the set A.

**Definition 1.17** **The crisp set is a group of objects (say U) that have the similar countability and finiteness qualities. A crisp set 'A' is a collection of items over the universal set U, where a random element can or cannot be a part of A.**

**Definition 1.18** Membership function $\nu_A$ in crisp set maps whole members in universal set X to set [0,1].

$$\nu_A : X \to \{0, 1\}$$

In fuzzy sets, each elements is mapped to [0,1] by membership function.

$$\nu_A : X \to [0, 1]$$

where [0,1] means real numbers between 0 and 1 (including 0,1). Consequently, fuzzy set is **vague boundary set** comparing with crisp set.

**Definition 1.19** If there are a universal set and a crisp set, we consider the set as a subset of the universal set. In the same way, we regard a fuzzy set $A$ as a subset of universal set $X$

**Definition 1.20** The $\alpha - cutsetA_\alpha$ is made up of members whose membership is not less than $\alpha$.

$$A_\alpha = \{x \epsilon X | \nu_A(x) \geq \alpha\}$$

note that $\alpha$ is arbitrary. This $\alpha$-cut set is a crisp set.

**Definition 1.21** Suppose there are two fuzzy sets A and B. When their degrees of membership are same, we say "A and B are equivalent". That is,

$$A = B \text{ iff } \nu_A(x) = \nu_B(x), \text{ for all } x \epsilon X$$

If $\nu_A(x) \neq \nu_B(x)$ for any element, then $A \neq B$. If the following relation is satisfied in the fuzzy set $A$ and $B$, $A$ is a **subset** of B.

$$\nu_A(x) \leq \nu_B(x), forall x \epsilon X$$

**Definition 1.22** The relation $A \underline{c} B$, We call that A is a subset of B. In addition, if the relation holds, A is a **proper subset of** $B$.

$$\nu_A(x) < \nu_B(x), \text{ for all } x \epsilon X$$

This relation can be written as

$$AcB \text{ iff } A\underline{c}B \text{ and } A \neq B.$$

**Definition 1.23** We can find complement set of fuzzy set A likewise in crisp set. We denote the complement set of A as $A'$ . Membership degree can be calculated as following

$$. \ \nu'_A(x) = 1 - \nu_A(x), \text{ for all } x \epsilon X.$$

If we calculate the complement set of "adult" as $A'$ , we may have

A' = (5, 1), (15, 0.9), (25, 0.1).

**Definition 1.24** consider a fuzzy set satisfying $A \neq \phi$ and $A \neq X$... The pair ( A , A' ) is defined as **fuzzy partition.**

**Definition 1.25** **Intersection** of fuzzy sets A and B takes smaller value of membership function between A and B.

$$\nu_A(x) = Min[\nu_A(x), \nu_B(x)], forall x \epsilon X$$

.

Intersection $A \cap B$ is a subset of A or B.

**Definition 1.26** A fuzzy number is a generalization of a regular real number in the sense that it does not refer to one single value but rather to a connected set of possible values, where each possible value has its own weight between 0 and 1.This weight is called **the membership function.**

**Definition 1.27** The degree of fuzziness is determined by the a-cut which is called the **fuzzy spread.**

# Chapter 2

# Fuzzy Arithmetic

## 2.1 Introduction

Fuzzy Arithmetic Operations are Exact Mathematical Methods, introduced by Zadeh (1975).Fuzzy numbers are specific types of fuzzy sets that are used for representing the values of real-world parameters when exact values are not measurable due to incomplete information or a lack of knowledge. Fuzzy number: $A : R \to [0, 1]$ A fuzzy set A in R is said to be a fuzzy number if i) A must be a normal fuzzy set ii) A must be a closed interval for every $\alpha\epsilon(0, 1]$ iii) A must be bounded

## 2.2 Fuzzy Number

A fuzzy number is a special case of a convex, normalized fuzzy set of the real line. Just like fuzzy logic is an extension of Boolean logic (which uses absolute truth and falsehood only, and nothing in between), fuzzy numbers are an extension of real numbers. Calculations with fuzzy numbers allow the incorporation of uncertainty on parameters, properties, geometry, initial conditions, etc. The arithmetic calculations on fuzzy numbers are implemented using fuzzy arithmetic operations, which can be done by two different approaches: (1) interval arithmetic approach; and (2) the extension principle approach. A fuzzy number is equal to a fuzzy interval.

## 2.3 Linguistic Variables

The fuzzy numbers represent linguistic concepts such as very small medium and so on.A base variable is a variable in a classical sense represented by any physical variable (e.g. temperature, pressure, speed, voltage, etc) as well as any numerical variables (e.g. age interest, rate, performance, salary, blood count, etc.) A variable whose states are fuzzy numbers assigned to relevant linguistic terms. The idea of linguistic variables is essential to development of the fuzzy set theory. Fuzzy logic is primarily associated with quantifying and reasoning out imprecise or vague terms that appear in our languages. These terms are referred to as linguistic or fuzzy variables. A fuzzy system is any system whose variables (or, at least, some of them) range over states that are fuzzy numbers rather than real numbers. These fuzzy numbers may represent linguistic terms such as "very small," "medium," and so on, as interpreted in a particular context. If they do, the variables are called linguistic variables.

Each linguistic variable is defined in terms of a base variable, whose values are real numbers within a specific range. A base variable is a variable in the usual sense, as exemplified by any physical variable (e.g., temperature, pressure, electric current, magnetic flux, etc.) as well as any other numerical variable (e.g., interest rate, blood count, age, performance, etc.). In a linguistic variable, linguistic terms representing approximate values of a base variable, relevant to a particular application, are captured by approximate fuzzy numbers. That is, each linguistic variable consists of the following elements:

- A name, which should capture the meaning of the base variable involved
- A base variable with its range of values (a closed interval of real numbers)
- A set of linguistic terms that refer to values of the base variable
- A semantic rule, which assigns to each linguistic term its meaning—an appropriate fuzzy number defined on the range of the base variable

An example of a linguistic variable is shown in figure. Its name is "performance," which captures the meaning of the associated base variable—a variable that expresses the performance (in percentage) of a goal-oriented entity (a person, machine, organization, method, etc.) in some context by real numbers in the interval [0,100], Linguistic values (states) of the linguistic variable are "very small," "small," "medium," "large," and "very large." Each of these linguistic terms is assigned one of the trapezoidal-shaped fuzzy numbers by a semantic rule. Examples of linguistic variables, example 1 and example 2 are given in figure 2.1 and figure 2.2.

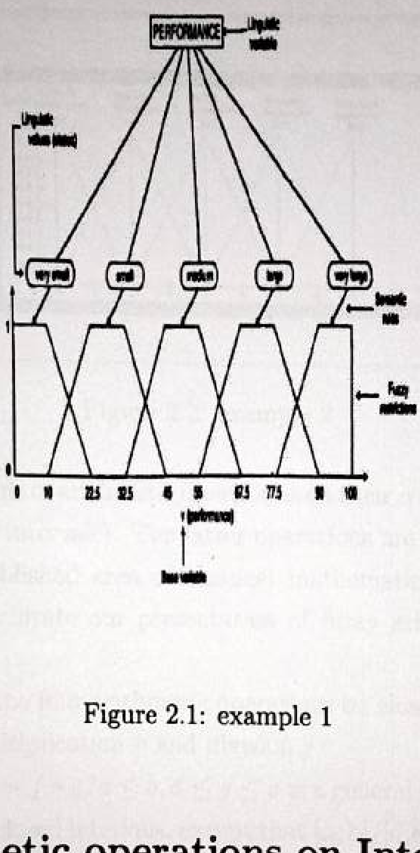


Figure 2.1: example 1

## 2.4 Arithmetic operations on Intervals

Fuzzy arithmetic is based on two properties of fuzzy numbers: (1) each fuzzy set, and thus also each fuzzy number, can fully and uniquely be represented by its $\alpha$-cuts and $\alpha$ - cuts of each fuzzy number are closed intervals of real numbers for all $\alpha \epsilon (0, 1]$.These properties enable us to define arithmetic operations on

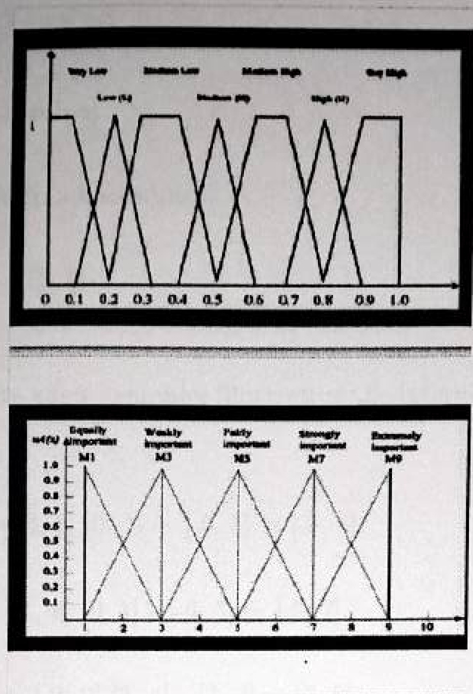


Figure 2.2: example 2

fuzzy numbers in terms of arithmetic operations on their $\alpha$-cuts (i.e., arithmetic operations on closed intervals). The latter operations are a subject of interval analysis, a well-established area of classical mathematics;we overview them in this section to facilitate our presentation of fuzzy arithmetic in the next section.

Let * denote any of the four arithmetic operations on closed intervals addition +, subtraction -, multiplication • and division /

Then $[a, b] * [d, e] = f * g/a \leq b, d \leq g \leq e$ is a general property of all arithmetic operations on closed intervals, except that [a, b]/[d,e] is not defined,when $0 \epsilon [d, e]$

The four arithmetic operations of closed intervals are defined as follows:

i) [a,b]+[d,e]=[a+d,b+e]

ii) [a,b]-[d,e]=[a+d,b-e]

iii) [a,b].[d,e]=[min(ad,ae,bd,be)]

iv) $\frac{[a,b]}{[d,e]} = [a, b] - [\frac{1}{e}, \frac{1}{d}]$

The following are a few examples illustrating the interval-valued arithmetic operations:

[2, 5] + [1, 3] = [3, 8] [0, 1] + [-6, 5] = [-6, 6],

[2, 5] - [1, 3] = [-1, 4] [0, 1] - [-6, 5] = [-5, 7],

[-1, 1] [-2, -0.5] = [-2, 2] [3, 4] - [2, 2] = [6, 8],

[-1,11] [-2, -0.5] = [-2, 2) [4, 10] / [1, 2] = [2,10] .

Arithmetic operations on closed intervals satisfy some useful properties. To overview them,

let A = [a1, a2], B = [b1, b2], C = [c1, c2], 0 = [0, 0],1 = [1, 1].

Using these symbols, the properties are formulated as follows:

1. A+B=B+A,
A.B = B . A (commutativity).

2. (A+B)+C=A+(B+C)
(A B) C = A (B C) (associativity).

3. A=O+A=A+O
A = 1.A = A.1 (identity)

Arithmetic operations on Intervals are shown in figure 2.3a,figure 2.4b and 2.5c.

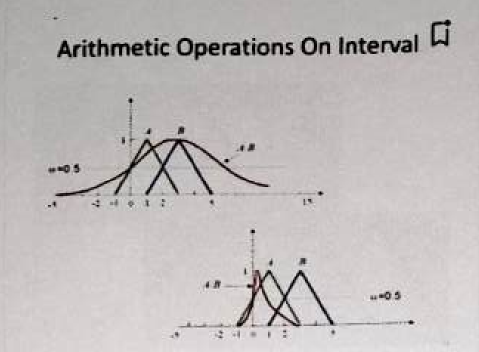


Figure 2.3: a

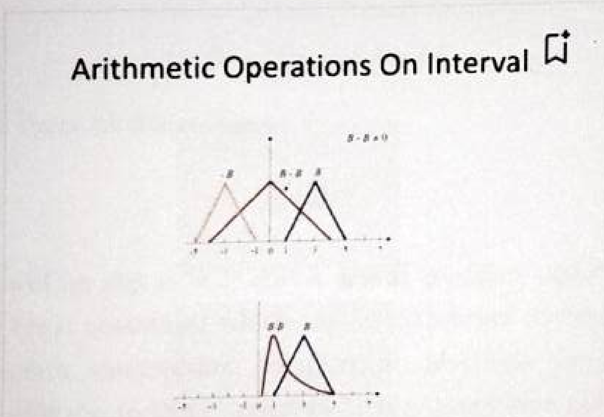


Figure 2.4: b

## 2.5 Arithmetic Operations on Fuzzy Numbers

Let A and B denote Fuzzy numbers and let * denote any of the four basic arithmetic operations.

Define $(A * B)^{\alpha} = A^{\alpha} * B^{\alpha}$ for any $\alpha \epsilon [a, b]$

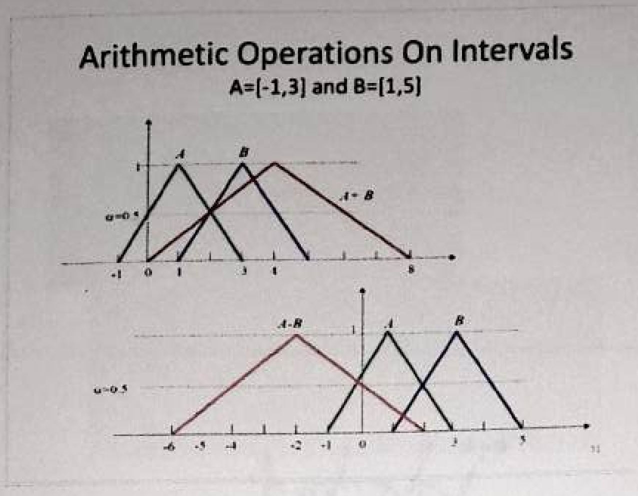


Figure 2.5: c

Then

$$A * B = U_{\alpha\epsilon[0,1]\alpha}(A * B)$$

Note: A*B is a fuzzy numbers, since

$$(A * B)^{\alpha}$$

is a closed interval for any $\alpha\epsilon[0,1]$ and A and B are fuzzy number.

Among the basic operations which can be performed on fuzzy sets are the operations of union, intersection, complement, algebraic product and algebraic sum. In addition to these operations, new operations called "bounded-sum" and "bounded-difference" were defined by L. A. Zadeh to investigate the fuzzy reasoning which provides a way of dealing with the reasoning problems which are too complex for precise solution. This paper investigates the algebraic properties of fuzzy sets under these nex operations of bounded-sum and bounded-difference and the properties of fuzzy sets in the case where these new operations are combined with the well-known operations of union, intersection, algebraic product and algebraic sum. Arithmetic Operations on fuzzy numbers are shown in figure 2.6a and figure 2.7b

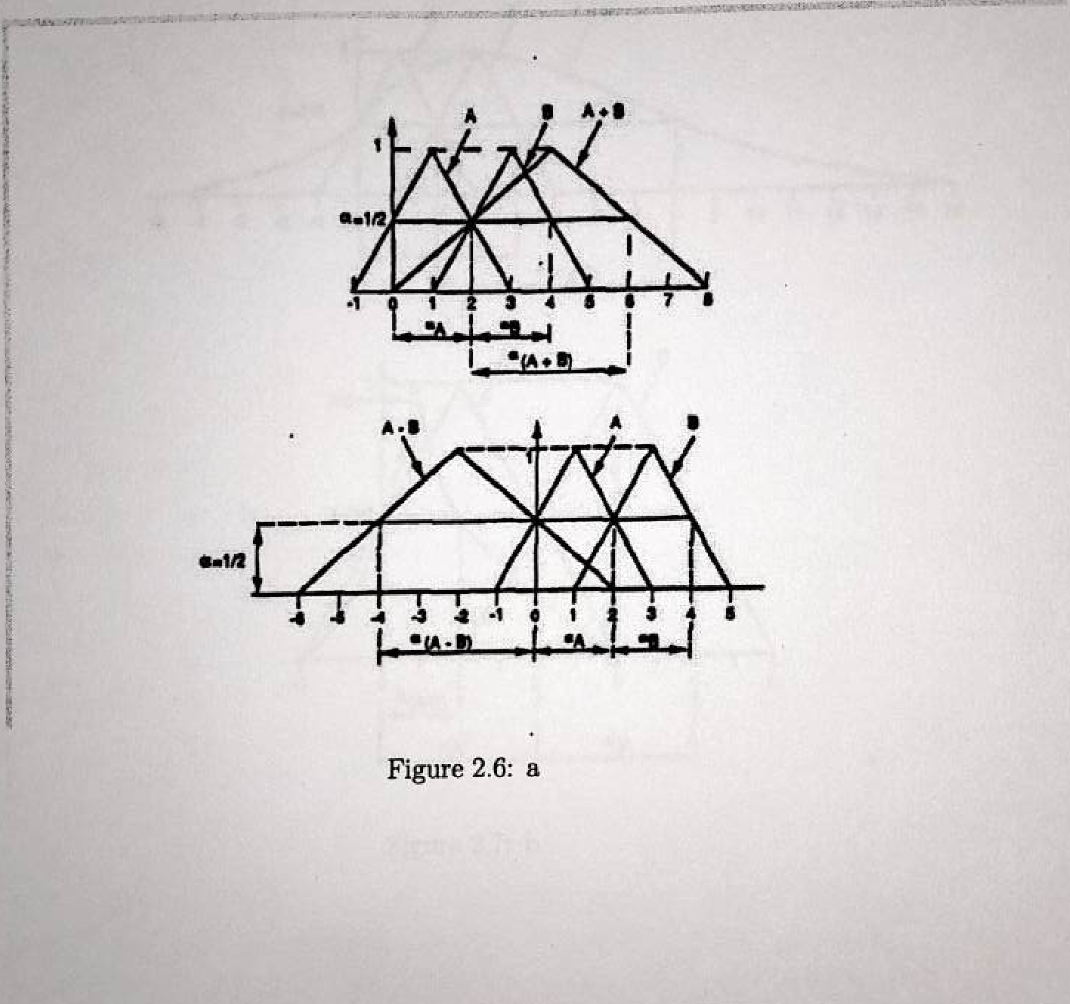


Figure 2.6: a

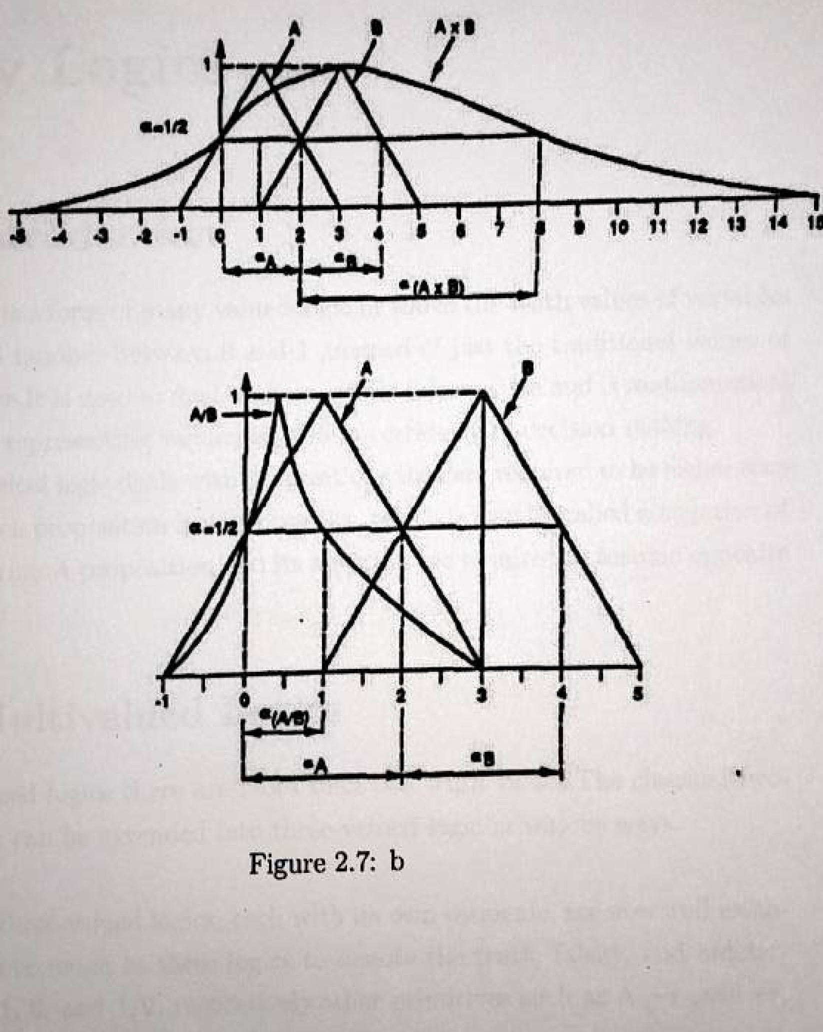


Figure 2.7: b

# Chapter 3

# Fuzzy Logics

## 3.1 Introduction

Fuzzy logic is a form of many valued logic in which the truth values of variables may be real number between 0 and 1 ,instead of just the traditional values of truth or false.It is used to deal with uncertain information and is mathematical method for representing vagueness and uncertainity in decision making.

Classical logic deals with propositions that are required to be either true or false. Each proposition has its opposite, which is usually called a negation of the proposition.A proposition and its negation are required to assume opposite truth values.

## 3.2 Multivalued Logics

In multivalued logics there are more than two truth values.The classical two-valued logic can be extended into three-valued logic in various ways.

Several three-valued logics, each with its own rationale, are now well established. It is common in these logics to denote the truth, falsity, and indeterminacy by 1, 0, and 1/2, respectively.other primitives such as $\wedge$ ,$\rightarrow$ ,and $\leftrightarrow$, differ from one three -valued logic to another.

Once the various three-valued logics were accepted as meaningful and useful, it became desirable to explore generalizations into n-valued logics for an arbitrary number of truth values . Several n-valued logics were, in fact, developed in the 1930s. For any given n, the truth values in these generalized logics are usually labelled by rational numbers in the unit interval [0, 1]. These values are obtained by evenly dividing the interval between 0 and 1 are exclusive. The set $T_n$ of truth values of an n-valued logic is thus defined as

$$T_n = [0 = \frac{0}{n-1}, \frac{1}{n-1}, \frac{2}{n-1}, \ldots, \frac{n-2}{n-1}, \frac{n-1}{n-1} = 1]$$

These values can be interpreted as degrees of truth.

The first series of n-valued logics for which n greater than or equal to 2 was proposed by Lukasiewicz in the early 1930s as a generalization of his three-valued logic. It uses truth values in T and defines the primitives by the following equations:

$$a \wedge b = \min\ (a,b)$$
$$a \vee b = \max\ (a,b)$$
$$a \rightarrow b = \min\ (1,1+b-a)$$

## 3.3 Linguistic Hedges

Linguistic hedges (or simply hedges) are special linguistic terms by which other linguistic terms are modified. Linguistic terms such as very, more or less, fairly, or extremely are examples of hedges. They can be used for modifying fuzzy predicates, fuzzy truth values, and fuzzy probabilities. For example, the proposition "x is young," which is assumed to mean "x is young is true," may be modified by the hedge the following three ways:

"x is very young is true,"

"x is young is very true,"

"x is very young is very true.

Similarly, the proposition "x is young is likely" may be modified to "x is young is very likely," and so forth.

In general, given a fuzzy proposition

p : x is F

and a linguistic hedge, H, we can construct a modified proposition,

$$\text{Hp : x is HF,}$$

where HF denotes the fuzzy predicate obtained by applying the hedge H to the given predicate F. Additional modifications can be obtained by applying the hedge to the fuzzy truth value or fuzzy probability employed in the given proposition.

It is important to realize that linguistic hedges are not applicable to crisp predicates, truth values, or probabilities. For example, the linguistic terms very horizontal, very pregnant, very teenage, or very rectangular are not meaningful. Hence, hedges do not exist in classical logic.

Any linguistic hedge H, may be interpreted as a unary operation h ,on the unit interval [0,1].For example, the hedge 'very' is often interpreted as the unary operation h(a)= $a^2$ while the hedge fairly is interpreted as

h(a)= $\sqrt{a}(a\epsilon[0,1])$ .Let unary operations that represent Linguistic hedges be called modifiers.

Given a fuzzy predicate F on X and a modifier h that represents a linguistic hedge H, the modified fuzzy predicate HF is determined for each xEx by the equation

$$\text{HF(x)= h((F(x))}$$

This means that properties of linguistic hedges can be studied by studying properties of the associated modifiers.

In representing modifiers of linguistic hedges, we should avoid various ambiguities of natural language. For example, the linguistic term not very may be viewed as the negation of the hedge very, but it may also be viewed (as some authors argue) as a new hedge that is somewhat weaker than the hedge very. In our further considerations, we always view any linguistic term not H, where H is an arbitrary hedge, as the negation of H.

## 3.4 Fuzzy Quantifiers

The generic term fuzzy quantifier denote the collection of quantifiers in natural languages whose representative elements are: several, most, much, not many, very many, not very many, few, quite a few, large number, small number, close to five, approximately ten, frequently, etc. In our approach, such quantifiers are treated as fuzzy numbers which may be manipulated through the use of fuzzy arithmetic and, more generally, fuzzy logic.
we can extend the scope of the fuzzy predicates by the use of fuzzy quantifiers. In general, fuzzy quantifiers are fuzzy numbers that take part in fuzzy propositions.
Fuzzy qualification has the following forms

### Fuzzy Qualification Based on Truth

It claims the degree of truth of a fuzzy proposition.
**Expression :**It is expressed as x is t. Here, t is a fuzzy truth value.
**Example :** (Car is black) is NOT VERY True.

### Fuzzy Qualification Based on Probability

It claims the probability, either numerical or an interval, of fuzzy proposition.
**Expression :** It is expressed as x is . Here, is a fuzzy probability.
**Example :** (Car is black) is Likely.

### Fuzzy Qualification Based on Possibility

It claims the possibility of fuzzy proposition.
**Expression :**It is expressed as x is . Here, is a fuzzy possibility.
**Example :** (Car is black) is Almost Impossible..

Fuzzy quantifiers are of two kinds.

- Absolute quantifiers

- Relative quantifiers

### 3.4.1 Absolute quantifiers

Absolute quantifiers are defined on R and characterize linguistic terms such as about 5, much more that 50, at least about 10 and so on.
There are two basic forms of propositions that contains absolute fuzzy quantifiers.

#### First Form

The proposition

$$p: There\ are\ Q\ is\ in\ I\ such\ that\ V\ is\ F$$

where V is a variable that for each individual i in a given set I assumes a value V(i), F is a fuzzy set defined on the set of values of variable V, and Q is a fuzzy number on R. In general, I is an index set by which distinct measurements of variable V are distinguished.

**Example 3.1** "There are about 15 students in a given class whose fluency in English is high." Given a set of students, I, the value V(i) of variable V represents in this proposition the degree of fluency in English of student i (expressed, e.g., by numbers in [0, 1]), F is a fuzzy set defined on the set of values of variable V that expresses the linguistic term high, and Q is a fuzzy number expressing the linguistic term about 15.The example of first form is shown in figure 3.1 .

#### Second form

Fuzzy quantifiers of the first kind may also appear in fuzzy propositions of the form (second form)

$$p: There\ are\ Q\ is\ in\ I\ such\ that\ V1(i)\ is\ F1\ and\ V2(i)\ is\ F2,$$

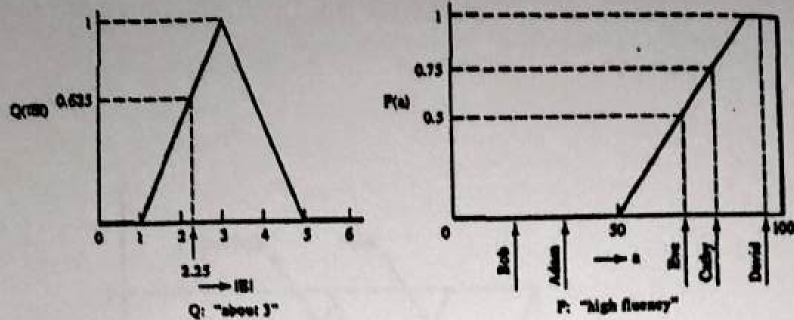


Figure 3.1:

where. V1, V2 are variables that take values from sets V1, V2, respectively, I is an index set by which distinct measurements of variables V1, V2 are identified (e.g., measurements on a set of individuals or measurements at distinct time instants), Q is a fuzzy number on R, and F1, F2 are fuzzy sets on V1, V2, respectively.

**Example 3.2** An example of second form of a quantified fuzzy proposition is the proposition "There are about 10 students in a given class whose fluency in English is.high and who are young." In this proposition,I is an index set by which students in the given class are labelled, variables V1 and V2 characterize fluency in English and age of the students, Q is a fuzzy number that captures the linguistic term "about 10 ," and F1, F2 are fuzzy sets that characterize the linguistic terms "high" and "young", respectively.The example is shown in figure 3.2

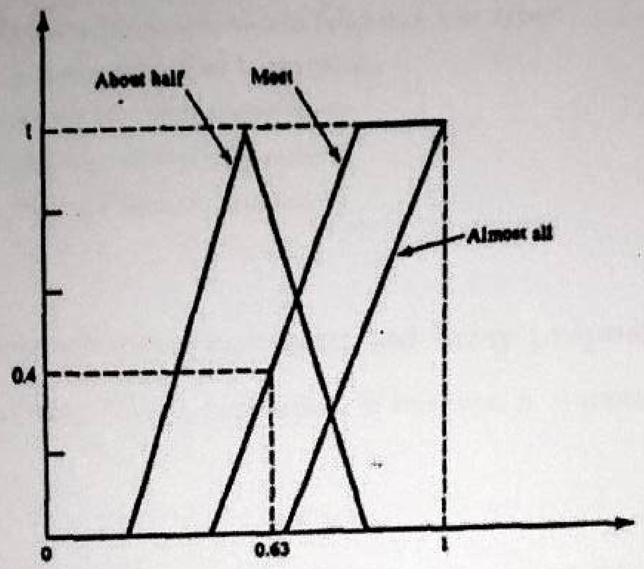


Figure 3.2:

### 3.4.2 Relative quantifiers

Relative quantifiers are represented by fuzzy numbers on the unit interval [0, 1] and characterize linguistic terms such as "almost all," "about half," "most," and so on.
Fuzz propositions with quantifiers of the second kind have the general form

p : Among is in I such that V1(i) is F1 there are Qis in I such that V2(i) is F2

where Q is a fuzzy number on [0, 1], and the meaning of the remaining symbols is the same as previously defined.

## 3.5 Fuzzy prepositions

The fundamental difference between classical propositions and fuzzy propositions is in the range of their truth values. While each classical proposition is required to be either true or false, the truth or falsity of fuzzy propositions is a matter of degree.

We classify fuzzy prepositions into following four types:

1.unconditional and unqualified prepositions

2.unconditional and qualified prepositions

3.conditional and unqualified prepositions

4.conditional and qualified prepositions

### 3.5.1 unconditional and qualified fuzzy prepositions

The canonical form of fuzzy propositions of this type, p, is expressed by the sentence

$$p: V \text{ is } F$$

where V is a variable that takes values v from some universal set V, and F is a fuzzy set on V that represents a fuzzy predicate, such as tall, expensive, low, normal, and so on. Given a particular value of V (say, v), this value belongs to F with membership grade F(v). This membership grade is then interpreted as the degree of truth, T (p), of proposition p. That is,

$$T(p)= F(v)$$

for each given particular value v of variable V in proposition p. This means that T is in effect a fuzzy set on [0, 1], which assigns the membership grade F(v) to each value v of variable V.

Consider, for example, that I is a set of persons, each person is characterized by his or her Age, and a fuzzy set expressing the predicate Young is given. Denoting our variable by Age and our fuzzy set by Young, we can exemplify the general form by the specific fuzzy preposition p : Age(i) is Young.

The degree of truth of this proposition, T (p), is then determined for each person i in I via the equation

T(p) =Young (Age(i))

### 3.5.2 unconditional and qualified prepositions

Propositions p of this type are characterized by either the canonical form

P: V is F is S

or the canonical form

P: pro V is F is P

where V and F have the same meaning as in the previous type , Pro (V is F) is the probability of fuzzy event "V is F," S is a fuzzy truth qualifier, and P is a fuzzy probability qualifier. If desired, V may be replaced with V(i), which has the same meaning as in previous type say that the proposition P: V is F is S is truth-qualified, while the proposition P: pro V is F is P ,is probability qualified. Both S and P are represented by fuzzy sets on [0,1].
The degree of truth, T (p), of any truth-qualified proposition p is given for each $v \epsilon V$by the equation

T(p) = S(F(v)).

### 3.5.3 conditonal and unqualified prepositions

Propositions p of this type are expressed by the canonical form

P: if x is A ,then y is B

where x,y are variables whose values are in sets X, Y, respectively, and A, B are fuzzy sets on X, Y, respectively. These propositions may also be viewed as propositions of the form (x,y) is R ,

where R is a fuzzy set on X x Y that is determined for each $x \epsilon X$ and each $y \epsilon Y$ by the formula

R(x,y) = j [A(x),B(y)]

where a denotes a binary operation on [0, 1] representing a suitable fuzzy implication.

### 3.5.4 conditonal and qualified prepositions

Propositions of this type can be characterized by either the canonical form

P: If x is A,then y is B is S

or the canonical form

P: Pro x is A$||yisBisP$,

where Pro X is A—y is B is -a conditional probability.

Since methods introduced for the other types of propositions can be combined to deal with propositions of this type, we do not deem it necessary to discuss them further.

# Chapter 4

# Application Of Fuzzy Mathematics

## 4.1 Introduction

In the natural sciences, social sciences, engineering and technology in various fields, will involve a large number of fuzzy factors and fuzzy information processing problem, fuzzy technology into almost all areas, the column has a larger international conference on the topic of more than a dozen fuzzy year, various fuzzy technological achievements and products have gradually blurred by the laboratory to society, some have achieved remarkable social and economic benefits such as metallurgy, machinery, petroleum, chemical, electric power, electronics, light industry, energy, transportation, health care, health, agriculture, forestry, geography, hydrology, seismology, meteorology, environmental protection, construction, behavioral science, management science, law, education, military science and so on, each area has its own examples of successful applications.

## 4.2 Fuzzy Clustering

Clustering is one of the most fundamental issues in pattern recognition. It plays a key role in searching for structures in data. Given a finite set of data, X, the problem of clustering in X is to find several cluster centers that can properly characterize relevant classes of X. In classical cluster analysis, these classes are required to form a partition of X such that the degree of association is strong for data within blocks of the partition and weak for data in different blocks. However, this requirement is too strong in many practical applications, and it is thus desirable to replace it with a weaker requirement. When the requirement of a crisp partition of X is replaced with a weaker requirement of a fuzzy partition or a fuzzy pseudopartition on X, we refer to the emerging problem area as fuzzy clustering.For example :In fuzzy clustering, data points can potentially belong to multiple clusters. For example, an apple can be red or green (hard clustering), but an apple can also be red AND green (fuzzy clustering). Here, the apple can be red to a certain degree as well as green to a certain degree. Instead of the apple belonging to green [green = 1] and not red [red = 0], the apple can belong to green [green = 0.5] and red [red = 0.5]. These value are normalized between 0 and 1; however, they do not represent probabilities, so the two values do not need to add up to 1. Fuzzy pseudopartitions are often called fuzzy c partitions, where c designates the number of fuzzy classes in the partition.

There are two basic methods of fuzzy clustering. One of them, which is based on fuzzy c-partitions, is called a fuzzy c-means clustering method. The other method, based on fuzzy equivalence relations, is called a fuzzy equivalence relation-based hierarchical clustering method. We describe basic characteristics of these methods and illustrate each of them by an example. Various modifications of the described methods can be found in the literature.

### 4.2.1 Fuzzy c-Means Clustering Method

Fuzzy c-means (FCM) is a data clustering technique in which a data set is grouped into N clusters with every data point in the dataset belonging to every cluster to a certain degree. For example, a data point that lies close to the center of a cluster will have a high degree of membership in that cluster, and another data point that lies far away from the center of a cluster will have a low degree of membership to that cluster

Let $X = x_1, x_2, ..., x_n$be a set of given data. A fuzzy pseudopartition or fuzzy c-partition of X is a family of fuzzy subsets of X, denoted by $P = (A_1, A_2, ..., A_{c},)$ which satisfies

$$\Sigma_{i=1}^{c} A_i(x_k) = 1$$

for all $k \epsilon N_e$ and

$$0 < \Sigma_{k=1}^{n} A_i(x_k) < n$$

for all $i \epsilon N_c$, where $c$ is a positive integer.
For instance, given $X = x_1, x_2, x_3$ and

$$A_1 = .\frac{6}{x_1} + \frac{1}{x_2} + .\frac{1}{x_3}$$

,

$$A_2 = .\frac{4}{x_1} + \frac{0}{x_2} + .\frac{9}{x_3},$$

then$A_l$, $A_2$is a fuzzy pseudopartition or fuzzy 2-partition of $X$. Fuzzy quantizations (or granulations) of variables in fuzzy systems are also examples of fuzzy pseudopartitions.

The problem of fuzzy clustering is to find a fuzzy pseudopartition and the associated cluster centers by which the structure of the data is represented as best as possible. This requires some criterion expressing the general idea that associations (in the sense described by the criterion) be strong within clusters and weak between clusters. To solvė the problem of fuzzy clustering, we need to formulate this criterion in terms of a performance index. Usually, the performance index is based upon cluster centers. Given a pseudopartition $T = A_1, A_2, ..., A_c$, the c cluster centers, $v_1, v_2, ..., v_c$. associated with the partition are calculated by the formula

$$V_i = \frac{\Sigma_{k=1}^{n} [A_i(X_k)]^m X_k}{\Sigma_{k=1}^{n} [A_i(X_k)]^m}$$

for all $i \epsilon N_n$, where $m > 1$ is a real number that governs the influence of membership grades.

### 4.2.2 Fuzzy c- Means Algorithm

This algorithm works by assigning membership to each data point corresponding to each cluster center on the basis of distance between the cluster center and the data point. More the data is near to the cluster center more is its membership towards the particular cluster center. Clearly, summation of membership of each data point should be equal to one. After each iteration membership and cluster centers are updated according to the formula:

$$\mu_{ij} = \frac{1}{\Sigma_{k=1}^{c}(\frac{d_{ij}}{d_{ik}})^{(\frac{2}{m-1})}}$$

Let $X = x_1, x_2, x_3..., x_n$ be the set of data points and $V = v_1, v_2, v_3..., v_c$ be the set of centers.

1. Randomly select 'c' cluster centers
2. Calculate the fuzzy membership $ij$ using:

$$\mu_{ij} = \frac{1}{\Sigma_{k=1}^{c}(\frac{d_{ij}}{d_{ik}})^{(\frac{2}{m-1})}}$$

3. Compute the fuzzy centers $v_i$ using:

$$V_i = \frac{\Sigma_{k=1}^{n}[A_i(X_k)]^m X_k}{\Sigma_{k=1}^{n}[A_i(X_k)]^m}$$

4. Repeat step 2 and 3 until the minimum i value is achieved.

**Example**

To illustrate the fuzzy c-means algorithm, let us consider a data set X that consists of the following 15 points in $R^2$

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $x_k1$ | 0 | 0 | 0 | 1 | 1 | 1 | 2 | 3 | 4 | 5 | 5 | 5 | 6 | 6 | 6 |
| $x_k2$ | 0 | 2 | 4 | 1 | 2 | 3 | 2 | 2 | 2 | 1 | 2 | 3 | 0 | 2 | 4 |

Assume that we want to determine a fuzzy pseudopartition with two clusters (*i.e.*, $c = 2$). Assume further that we choose $m = 1.25, s = 0.01; \left|\left| \bullet \right|\right|$ is the Euclidean distance, and the initial fuzzy pseudopartition is $\rho^{(0)} = A_1, A_2$ with

$$A_1 = \frac{.854}{x_1} + \frac{.854}{x_2} + \ldots + \frac{.854}{x_15},$$

$$A_2 = \frac{.146}{x_1} + \frac{.146}{x_2} + \ldots + \frac{.146}{x_15}$$

The table b gives the values 6and figure 4.1a shows the result

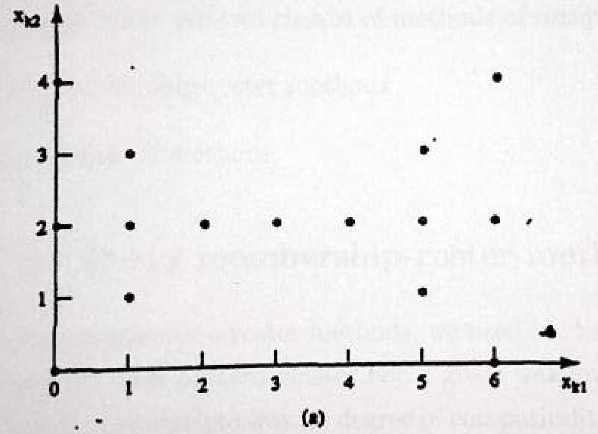


Figure 4.1: a

| k | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $A_1(x_k)$ | .99 | 1 | .99 | 1 | 1 | 1 | .99 | .47 | .01 | 0 | 0 | 0 | .01 | 0 | .01 |
| $A_2(x_k)$ | .01 | 0 | .01 | 0 | 0 | 0 | .01 | .53 | .99 | 1 | 1 | 1 | .99 | 1 | .99 |

(b)

Then, the algorithm stops for $t = 6$ and we obtain the fuzzy pseudopartition.The two cluster centers are $v_1 = (0.88, 2)$ and $v_2 = (5.14, 2)$.

## 4.3 Pattern Recognition

The capability of recognizing and classifying patterns is one of the most fundamental characteristics of human intelligence. As a field of study, pattern recognition has been evolving since the early 1950,,in close connection with the emergence and evolution of computer technology. Pattern Recognition may be defined as a process by which we search for structures in data and classify these structures in to categories such that the degree of association is high among structures of the same category and low among structures of different categories.The classification of objects in to categories is the subject of cluster analysis.There are two classes of methods of fuzzy pattern recognition.

1. Fuzzy membership-roster methods
2. Fuzzy syntactic methods

### 4.3.1 Fuzzy membership-roster method

In fuzzy membership-roster methods, we need to store only one standard pattern for each pattern classes.For a given unknown pattern, we measure, in an appropriate way,its degree of compatibility with each standard pattern and then classify the pattern to a particular class according to some criteria.

(a) Assume that $n$ pattern classes are recognized, which are labelled by the integers in $N_n$.
Given a relevant pattern

$$\mathbf{u} = < u_1, u_2, ...u_p >,$$

where$u_i$ is the measurement associated with the $i^{th}$ feature of the pattern $(i \epsilon N_p)$

(b) let $A_k(u)$ denote the degree of compatibility of u with the standard pattern representing class $k(k \epsilon N_n)$. A given pattern is usually classified by the largest value of $A_k(u)$ for all $k \epsilon N_n$ but other classification criteria have also been suggested.

Specific methods for selecting pattern features, determining the degrees $A_k(u)$, and classifying given patterns according to these degrees have been developed for specific types of pattern recognition problems

### 4.3.2 Fuzzy syntactic method

Classical syntactic methods of pattern recognition are based on the theory of formal languages and grammars. In these methods, pattern classes are represented by languages, each of which is a set of strings of symbols from a vocabulary that are generated by the pattern grammar. These methods are suitable for recognizing patterns that are rich in structural information which cannot be easily expressed in numerical values.

(a) Let V be a vocabulary, and let $V*$ denote the set of all strings formed by symbols from $V$, including the empty string.

(b) Then any subset of $V*$ is called a language based upon the vocabulary $V$. Some languages can be defined by grammars. A grammar, G, is the quadruple

$$G =< V_N, V_T, P, s >,$$

where

i. $V_N$ is a non terminal vocabulary;

ii. $V_T$ is a terminal vocabulary such that $V_r \cap V_N = o$;

iii. P is a finite set of production rules of the form $x \rightarrow y$, where $x$ and $y$ are strings of symbols from $V = V_N U V_T$ such that $x$ contains at least one symbol of $V_N$; and

iv. $s$ is the starting symbol ($s \epsilon V_N$)

(c) The language generated by grammar G, denoted by $L(G)$, is the set of all strings formed by symbols in $V_T$ that can be obtained by the production rules in $P$.

(d) In syntactic pattern recognition, features of patterns are represented by the elements of the terminal vocabulary $V_T$. They are usually called primitives. Each pattern is represented by a string of these primitives, and each pattern class is defined by a grammar that generates strings representing patterns in that class

(e) A fuzzy grammar, $FG$, is defined by the quintuple

$$\text{FG} = < V_n V_r, P, s, A >,$$

where

i. $V_N$ is a non terminal vocabulary

ii. $V_T$ is a terminal vocabulary

iii. $P$ is a finite set of production rules;

iv. $s$ is the starting symbol ($s \epsilon V_v$); and

v. $A$ is a fuzzy set defined on P

.

(f) The language generated by the fuzzy grammar is a fuzzy set, **L (FG)**, defined on the language generated by the associated crisp grammar. For each string $x \epsilon L(G)$,

$$L(FG)(x) = max \quad min A(p_i^k),$$

where $m$ is the number of derivations of string $x$ by grammar **FG**, $n_k$ is the length of the $k^{th}$ derivation chain, and $p_i^k$ denotes the $i^{th}$ production rule used in the $k^{th}$ derivation chain ($i = 1, 2, ..., n_k$).

## 4.4 Decision making

Making decisions is undoubtedly one of the most fundamental activities of human beings. The subject of decision making is,as the name suggests the study of how decisions are actually made and how they can be made better or more successfully. Applications of fuzzy sets within the field of decision making have,for the most part,consisted of fuzzifications of the classical theories of decision making. A decision is said to be made under conditions of certainity when the outcome for each action on be determined and ordered precisely. The problem cases are referred to as *individual decision making* and *multiperson decision making* respectively

### 4.4.1 Individual Decision Making

Fuzziness can be introduced in to the existing models of decision models in various ways. A decision situation in this model is characterized by the following components

1. a set A of possible actions
2. a set of goals $G_i$ $(i \epsilon N_n)$ each of which is expressed in terms of a fuzzy set defined on A
3. a set of constraints $C_j$ $(j \epsilon N_m)$ each of which is expressed in terms of a fuzzy set defined on A
4. Let $G'_i$ and $C'_j$ be fuzzy sets defined on sets $X_i$ and $Y_i$ respectively where $i \epsilon N_n$ and $j \epsilon N_m$
5. Then for each $i \epsilon N_n$ and $j \epsilon N_m$. We describe the meanings of actions in set A in terms of sets $X_i$ and $Y_i$ by functions.

$$g_i : A \to X_i$$
$$c_j : A \to Y_j$$

and express goals $G_i$ and constraints $C_j$ by the composition of $g_i$ with $G'_i$ and composition of $c_j$ and $C'_j$

$$G_i(a) = G_i'(g_i(a))$$
$$C_j(a) = C_j'(c_j(a))$$
$$\text{for each } a \epsilon A$$

A fuzzy decision D

$$D(a) = min[\inf_{i \epsilon N_n} G_i(a), \inf_{j \epsilon N_m} C_j(a)]$$
$$\text{for all } a \epsilon A$$

Once a fuzzy decision has been arrived at, it may be necessary to choose the " best"single crisp alternative â from this fuzzy set. When A is defined on $R$,it is preferable to determine â by an appropriate defuzzification method.

### 4.4.2 Multiperson decision making

when decisions made by more than one person are modeled,two differences from the case of a single decision maker can be considered:first,the goals of the individual decision makers may differ such that each places a different ordering on the alternatives;second, the individual decision makers may have access to different information upon which to base their decision.

1. here, each member of a group of $n$ individual decision makers is assumed to have a reflexive,anti symmetric and transitive preference ordering $p_k, k \epsilon N_n$, which totally or partially orders a set X of alternatives.
2. In order to deal with the multiplicity of opinion evidenced in the group, the social preference $S$ may be defined as a fuzzy binary relation with membership grade function

   $$S : X * X \to [0, 1],$$

   which assigns the membership grade $S(x_i, x_j)$, indicating the degree of group preference of alternative $x_i$ over $x_j$.

   (a) One simple method computes the relative popularity of alternative $x_i$ over $x_j$ by dividing the number of persons preferring $x_i$ to $x_j$, denoted by $N(x_i, x_j)$, by the total number of decision makers,$n$.This scheme corresponds to the simple majority vote. thus,

$$S(x_i, x_j) = \frac{N(x_i,x_j)}{n}$$

(b) Other methods of aggregating the individual preferences may be used to accommodate different degrees of influence exercised by the individuals in the group.For instance, a dictatorial situation can be modeled by the group preference relation $S$ for which

$$S(x_i, x_j) = \begin{cases} 1, & if x_i >^k x_j \quad \textit{for some individual } k \\ 0, & otherwise, \end{cases}$$

where $>^k$ represents the preference ordering of the one individual $k$ who exercises complete control over the group decision.

3. Once the fuzzy relationship $S$ has been defined,the final non fuzzy group preference can be determined by converting $S$ into its resolution form

$$S = \bigcup_{\alpha \epsilon [0,1]} \alpha^{\alpha} S,$$

which is the union of the crisp relations $\alpha_S$ comprising the $\alpha$-cuts of the fuzzy relation S,each scaled by $\alpha$.Each value $\alpha$ essentially represents the level of agreement between the individuals concerning the particular crisp ordering $\alpha_S$

4. The largest value a for which the unique compatible ordering on $XxX$ is found represents the maximized agreement level of the group, and the crisp ordering itself represents the group decision.

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4. The largest value a for which the unique compatible ordering on $XxX$ is found represents the maximized agreement level of the group, and the crisp ordering itself represents the group decision.

## 4.5 Application Of Fuzzy In different fields

In terms of soft science, fuzzy technology has been used in the investment decision-making, corporate benefit assessment, regional development planning, economic macro-control, in areas such as long-term market forecasting fuzzy. Fuzzy theory will greatly promote scientific and quantitative research soft science. For example, Yamaichi Securities in Tokyo fuzzy logic system to manage large stocks have price assessment certificate, the system uses about one hundred rules to make buy and sell decisions.

In terms of earthquake science, fuzzy technology has been related to the long-term earthquake prediction, seismic hazard analysis and potential source identification, earthquake prediction and earthquake disaster reduction countermeasures and other fields.

In industrial process control, metallurgical furnaces has been achieved fuzzy control, fuzzy control chemical process, cement kiln, glass kiln fuzzy control, etc., fuzzy control technology has become an effective means of control of complex systems, greatly broadened the automatic control Application

In the appliance industry, there have been blurred washing machines, air conditioners fuzzy, fuzzy vacuum cleaners, auto-focus camera and camera blur, fuzzy control televisions, microwave ovens and other household appliances fuzzy, fuzzy logic control these appliances with a very low price varying degrees of change in performance, improve the machine's "IQ", popular in the consumer market.

In the field of artificial intelligence and computer tech, there have been a fuzzy inference engine, fuzzy control computer, fuzzy expert systems, fuzzy database fuzzy speech recognition system, graphics fuzzy character recognition systems, fuzzy robots and other high-tech products, but also the emergence of F -prolog, Fuzz-C and other language system.

In the aerospace and military fields, technology has been used in the aircraft fuzzy butt, C3I command automation systems, etc., such as NASA (NASA) is using fuzzy technology development between the shuttle and the space station as a navigation dock automation system.

# CONCLUSION

In this dissertation, we have a basic idea about fuzzy sets This dissertation consists of four chapters.Inthe Preliminary section, it consists of the basic idea that we already knows.The second and third chapters includes a very carefully crafted introduction to basic concepts of Fuzzy Arithmetics and Fuzzy Logics and the fourth chapter explains Applications of Fuzzy Mathematic.This dissertation gives a primary knowledge about Fuzzy Mathematic in the increasingly important ares.

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