STOCHASTIC PROCESS
PROJECT

Submitted to University of Kerala in partial fulfillment of the requirements for the award of the Degree of Bachelor of Science in Mathematics
by

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DECLARATION

We hereby declare that the work presented in this dissertation entitled **"STOCHASTIC PROCESS"** is based on the oriented work done by us under the supervision of *Mrs.Parvathy K Pillai*, Guest lecturer, Department of Mathematics & Statistics, All Saints' College Thiruvananthapuram and has not been submitted previously for the award of any degree.

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CERTIFICATE

This is to certify that the project entitled **STOCHASTIC PROCESS** Submitted to the University of Kerala,in partial fullfillment of the requirement of the FDP in Bachelor of Science(Mathematics) by

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is a bonafide work carried out by them under my supervision and it has not been submitted earlier or elsewhere for similar purposes according to best of my knowledge and belief.

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Project Guide

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Last, but not the least, we thank God Almighty for his helping hands.

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CONTENTS

PRELIMINARY

1.PROBABILITY:

Probability can be defined as the ratio of the number of favorable outcomes to the total number of outcomes of an event. For an experiment having 'n' number of outcomes, the number of favorable outcomes can be denoted by x.
Probability(Event) = Favorable Outcomes/Total Outcomes = x/n

2.PROBABILITY SPACE:

Probability space is the total number of possible ways the class of outcomes the probability of which one is attempting to determine, could possibly occur. For example, one can define a probability space which models the throwing of a die.

3.OUTCOME:

In probability theory, an outcome is a possible result of an experiment or trial. Each possible outcome of a particular experiment is unique, and different outcomes are mutually exclusive (only one outcome will occur on each trial of the experiment).

4.MEAN:

Mean is the arithmetical average of a set of values.Mean is the average of the given numbers and is calculated by dividing the sum of given numbers by the total number of numbers. Mean = (Sum of all the observations/Total number of observations)

5.VARIANCE:

Variance is a statistical measurement used to determine how far each number is from the mean and from every other number in the set.Variance is the expected value of the squared variation of a random variable from its mean value.

6.RANDOM VARIABLE:

A random variable is a variable whose value is unknown or a function that assigns values to each of an experiment's outcomes. A random variable can be either discrete i.e., having specific values. (eg: The number of cars sold by a car dealer in one month) or continuous i.e., any value in a continuous range. (eg: The amount of water in a 12-ounce bottle).

7.BINARY RANDOM VARIABLE:

A binary variable is a variable that has two possible outcomes.For example, a binary random variable can be used to represent the probability of a coin flip resulting in heads.

8.NORMAL DISTRIBUTION:

A continuous variable X having the symmetrical, bell shaped distribution is called a Normal Random Variable.The normal probability distribution (Gaussian distribution) is a continuous distribution which is regarded by many as the most significant probability distribution in statistics particularly in the field of statistical inference.

A continuous random variable X is said to follow Normal distribution if it's pdf is

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}, -\infty < x < \infty, -\infty < \mu < \infty, \sigma > 0$$

9.POISSON DISTRIBUTION:

A random variable X is said to follow the Poisson distribution if it's pdf is

$$f(x) = \frac{e^{-\lambda} * \lambda^x}{x!}, x = 0, 1, 2....$$

$$= 0, elsewhere$$

10.ARRIVAL RATE & INTER ARRIVAL TIME:

The arrival rate is the number of arrivals per unit of time. The inter arrival time is the time between each arrival into the system and the next.

Arrival rate = 1/Inter arrival time

11.EXPONENTIAL DISTRIBUTION:

In Probability theory and statistics, the exponential distribution is a continuous probability distribution that often concerns the amount of time until some specific event happens. It is a process in which events happen continuously and independently at a constant average rate. The exponential distribution has the key property of being memoryless. The exponential random variable can be either more small values or fewer larger variables.

The continuous random variable, say X is said to have an exponential distribution, if it has the following probability density function:

$$f(x;\lambda) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

$f(x;\lambda) =$ probability density function

λ = rate parameter

x = random variable

where, λ is called the distribution rate. The mean of the exponential distribution is $1/\lambda$ and the variance of the exponential distribution is $1/\lambda^2$.

12.CONDITIONAL DISTRIBUTION:

A conditional distribution is the probability distribution of a random variable, calculated according to the rules of conditional probability after observing the realization of another random variable.

13.MARKOV PROCESS:

A Markov chain or Markov process is a stochastic model describing a sequence of possible events in which the probability of each event depends only on the state attained in the previous event. A countably infinite sequence, in which the chain moves state at discrete time steps, gives a discrete-time Markov chain (DTMC). A continuous-time process is called a continuous-time Markov chain (CTMC).

14.OPERATIONS RESEARCH:

Operations research is an analytical method of problem-solving and decision-making that is useful in the management of organizations.

15.MATHEMATICAL MODELLING:

Mathematical modeling is a method that represents and explains real systems and occurrences using math formulas, descriptions and approaches.

Chapter-1

INTRODUCTION TO STOCHASTIC PROCESS

In probability theory and related fields, a stochastic or random process is a mathematical object usually defined as a sequence of random variables in a probability space, where the index of the sequence often has the interpretation of time. Stochastic processes are widely used as mathematical models of systems and phenomena that appear to vary in a random manner. Examples include the growth of a bacterial population, an electrical current fluctuating due to thermal noise, or the movement of a gas molecule.Stochastic processes have applications in many disciplines such as biology,chemistry,ecology, neuroscience,physics,image processing, signal processing,control theory,information theory, computer science,and telecommunications.Seemingly random changes in financial markets have motivated the extensive use of stochastic processes in finance.

The term random function is also used to refer to a stochastic or random process, because a stochastic process can also be interpreted as a random element in a function space. The terms stochastic process and random process are used interchangeably, often with no specific mathematical space for the set that indexes the random variables. But often these two terms are used when the random variables are indexed by the integers or an interval of the real line. If the random variables are indexed by the Cartesian plane or some higher-dimensional Euclidean space, then the collection of random variables is usually called a random field instead. The values of a stochastic process are not always numbers and can be vectors or other mathematical objects.

A stochastic or random process can be defined as a collection of random variables that is indexed by some mathematical set, meaning that each random variable of the stochastic process is uniquely associated with an element in the set. The set used to index the random variables is called the index set. Historically, the index set was some subset of the real line, such as the natural numbers, giving the index set the interpretation of time. Each random variable in the collection takes values from the same mathematical space known as the state space. This state space can be, for example, the integers, the real line or n-dimensional Euclidean space. An increment is the amount that a stochastic process changes between two index values, often interpreted as two points in time. A stochastic process can have many outcomes, due to its randomness, and a single outcome of a stochastic process is called, among other names, a sample function or realization.

Applications and the study of phenomena have in turn inspired the proposal of new stochastic processes. Examples of such stochastic processes include the Wiener process or Brownian motion process,used by Louis Bachelier to study price changes on the Paris Bourse, and the Poisson process, used by A. K. Erlang to study the number of phone calls occurring in a certain period of time.These two stochastic processes are considered the most important and central in the theory of stochastic processes,and were discovered repeatedly and independently.

Based on their mathematical properties, stochastic processes can be grouped into various categories, which include random walks,martingales,Markov processes, Lévy processes,Gaussian processes,random fields,renewal processes, and branching processes.The study of stochastic processes uses mathematical knowledge and techniques from probability, calculus, linear algebra, set theory, and topology as well as branches of mathematical analysis such as real analysis, measure theory, Fourier analysis, and functional analysis. The theory of stochastic processes is considered to be an important contribution to mathematics and it continues to be an active topic of research for both theoretical reasons and applications.

Stochastic processes can be indexed by discrete or continuous time, or even by other parameters. For example, a discrete-time stochastic process might represent measurements taken at regular intervals, while a continuous-time process might represent continuous observations.At each time index or point in the parameter space, a stochastic process assigns a random variable that represents the state of the system. These random variables can have various distributions, and their values are not deterministic but probabilistic. The random variables can be independent or dependent on each other. Dependence introduces memory into the process, meaning that the current state depends not only on the previous state but possibly on earlier states as well.

Chapter-2

TYPES OF STOCHASTIC PROCESS

2.1 BERNOULLI PROCESS

In probability and statistics, a Bernoulli process is a finite or infinite sequence of binary random variables, so it is a discrete-time stochastic process that takes only two values, canonically 0 and 1.

A coin flip is an example of a Bernoulli trial, which is any random experiment in which there are exactly two possible outcomes. The two possible outcomes of a Bernoulli trial are usually called success and failure. In this case, we define heads as a success and tails as a failure.

The component Bernoulli variables Yi are identically distributed and independent. Prosaically, a Bernoulli process is a repeated coin flipping, possibly with an unfair coin. Every variable Yi in the sequence is associated with a Bernoulli trial or experiment. They all have the same Bernoulli distribution. Much of what can be said about the Bernoulli process can also be generalized to more than two outcomes; this generalization is known as the Bernoulli scheme.

For any Bernoulli process Y_i we have,

$E(Y_i) = p$, $E(Y_i^2) = p$ and Var $(Y_i) = p(1-p)$

Let $(Y_i/i = 1, 2, ...)$be a Bernoulli process. Consider the sequence of partial sums (Sn); n = 1,2,...

where, $S_n = Y_1 + Y_2 + ... + Y_n$

$P(S_n = k/S_{n-1} = k) = p(Y_n = 0) = 1-p$ and $P(S_n = k/S_{n-1} = k-1) = p(Y_n = 1) = p$.

Chapter-2

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Then $S_n = S_{n-1} + Y_n$

Hence the probability of occurrence of S_n depends on $S_n - 1$. Therefore $S_n/n = 1, 2, \ldots$ is discrete state discrete parameter Markov process.

2.1.1 Binomial random process

The discrete state, discrete parameter Markov process, where each S, is a binomial random variable (for eg; Number of winning lottery tickets when you buy 10 tickets of the same kind. Also, the number of left-handers in a randomly selected sample of 100 unrelated people.) is called as a Binomial random process.

$P(S_n = k) = \binom{n}{k} p^k (1-p)^{n-k} E(S_n) = np$

Var $(S_n) = np(1-p) G_{S_n}(z) = (1 - p + pz)^n$

2.1.2 Non-homogeneous Bernoulli process

Let $(Y_i/i = 1, 2)$ be a Bernoulli process. If each Bernoulli variate i has distinct parameter p_i then the Bernoulli process is called as a non-homogeneous Bernoulli process. Yet another generalization of the Bernoulli process is to assume that each trial has more than 2 possible outcomes.

2.1.3 Random walk

The random walk process is an extension of the Bernoulli process. Here the process takes a step in the positive direction in the k^{th} trial if the outcome of the trial is a success and a step in the negative direction if the outcome is a failure.A typical example is the drunkard's walk, in which a point beginning at the origin of the Euclidean plane moves a distance of one unit for each unit of time, the direction of motion, however, being random at each step

Let $Y_i/i = 1, 2, \ldots$be a sequence of independent discrete r.v. and define the partial sum $Sn = \sum_{i=1}^{n} Yi$, Then the Markov chain $S_n/n = 1, 2, \ldots$is known as a random walk.

2.2 LEVY PROCESS

In probability theory, a Levy process, named after the French mathematician Paul Levy, is a stochastic process with independent, stationary increments: it represents the motion of a point whose successive displacements are random, in which displacements in pairwise disjoint time intervals are independent, and displacements in different time intervals of the same length have identical probability distributions. A Levy process may thus be viewed as the continuous-time analog of a random walk.

The most well known examples of Levy processes are the Wiener process, often called the Brownian motion process, and the Poisson process. Further important examples include the Gamma process, the Pascal process, and the Meixner process. Aside from Brownian motion with drift, all other proper Levy processes have discontinuous paths. All Levy processes are additive processes.

MATHEMATICAL DEFINITION:

A Levy process is a stochastic process $X = X_t : t \geq 0$ that satisfies the following properties:

1. $X_0 = 0$ almost surely;

2. Independence of increments: For any $0 \leq t_1 < t_2 < ... < tn < \infty$, $X_{t_2} - X_{t_1}, X_{t_3} - X_{t_2}, ..., X_{t_n} - X_{t_{n-1}}$ are mutually independent;

3. Stationary increments: For any $s < t, X_t - X_s$ is equal in distribution to X_{t-s} :

4. Continuity in probability: For any $\epsilon > 0$ and $t > 0$ it holds that $\lim_{h \to 0} P(|X(t+h) - X(t)| > \epsilon) = 0$.

If X is a Levy process then one may construct a version of X such that $t \to X_t$ is almost surely right continuous with left limits.

PROPERTIES:

Independent increments

A continuous-time stochastic process assigns a random variable X_t to each point $t \geq 0$ in time. In effect it is a random function of t. The increments of such a process are the differences $X_s - X_t$ between its values at different times $t \leq s$. To call the increments of a process independent means that increments $X_s - X_t$ and $X_u - X_v$ are independent random variables whenever the two time intervals do not overlap and, more generally, any finite number of increments assigned to pairwise non- overlapping time intervals are mutually (not just pairwise) independent.

Stationary increments

To call the increments stationary means that the probability distribution of any increment $X_t - X_s$ depends only on the length $t-s$ of the time interval; increments on equally long time intervals are identically distributed.

If X is a Wiener process, the probability distribution of $X_t - X_s$ is normal with expected value 0 and variance $t - s$.

If X is a Poisson process, the probability distribution of $X_t - X_s$ is a Poisson distribution with expected value

$$\lambda(t - s)$$

where $\lambda \geq 0$ is the "intensity" or "rate" of the process.

If X is a Cauchy process, the probability distribution of $X_t - X_s$ is a Cauchy distribution with density $f(x;t) = 1/\pi[t/(x^2 + t^2)]$

Infinite divisibility

The distribution of a Levy process has the property of infinite divisibility: given any integer n, the law of a Levy process at time t can be represented as the law of the sum of n independent random variables, which are precisely the increments of the Levy process over time intervals of length t/n, which are independent and identically distributed by assumptions 2 and 3. Conversely, for each infinitely divisible probability distribution F, there is a Levy process X such that the law of $X_1$is given by F.

Moments

In any Levy process with finite moments, the nth moment $\mu_n(t) = E(X_t^n)$is a polynomial function of t; these functions satisfy a binomial identity:
$\mu_n(t+s) = \sum_{k=0}^{n} \binom{n}{k} \mu_k(t)\mu_{n-k}(s)$.

2.3 POISSON RANDOM PROCESS

A Poisson process is a simple and widely used stochastic process for modeling the times at which arrivals enter a system. It is in many ways the continuous-time version of the Bernoulli process. For the Bernoulli process, the arrivals can occur only at positive integer multiples of some given increment size. The process by a sequence of independently and identically distributed binary random variables (IID random variables), Y1, Y2, . . . , where Yi = 1 indicates an arrival at increment i and Yi = 0 otherwise.

We observed that the process could also be characterized by the sequence of interarrival times. These interarrival times are geometrically distributed IID random variables. For the Poisson process, arrivals may occur at arbitrary positive times, and the probability of an arrival at any particular instant is 0. This means that there is no very clean way of describing a Poisson process in terms of the probability of an arrival at any given instant. It is more convenient to define a Poisson process in terms of the sequence of interarrival times, X1, X2, . . . , which are defined to be IID. Before doing this, we describe arrival processes in a little more detail.

Let N(0,1) represents the number of occurrences of a certain event in (0,1), then the discrete random process $(N(t) : t \geq (0,1)$ is a Poisson process provided the following postulates are satisfied

(i) P(1 occurrence in $(t, t+\Delta t)) = \lambda\Delta t + 0(\Delta t)$

(ii) P(0 occurrence in ($(t+\Delta t)) = 1 - \lambda\Delta t + 0(\Delta t)$

(iii) P(2 or more in $((t+\Delta t)) = 0(\Delta t)$

(iv) $N(r)$is independent of the number of occurrence of the event in any interval prior or after $(0, t)$

(v) The probability that the event occurs a specified number of times in $(fo, fo+1)$ depends only on but not on t_0

We know that the Poisson process is a continuous parameter discrete state process. Suppose that events occur successively in time so that intervals between successive events are independent and identically distributed according to an exponential distribution $F_X(x) = 1 - e^{-\lambda x}$ Let the number of events is the interval [0, 1] is denoted by N(t). Then the stochastic process $(N(t) : t \geq 0)$ is a Poisson process, with mean λ . Note that the number of events in the interval [0, 1] is a Poisson distribution with parameter λ.

The probability distribution of N(t) is given by, $P(N(t) = n) = \frac{e^{-\lambda t}(\lambda t)^n}{n!}, n = 0, 1, 2, ...$ This idea can be generalised to define the second order probability function of a homogeneous Poisson process:

PROPERTIES:

1. For each t, N(t) depends on λ and hence it is not a stationary stochastic process.

2. $E(N(t)) = Var(N(t)) = \lambda t$

$\lim_{t\to\infty} E(\frac{N(t)}{t}) = \lim_{t\to\infty} \frac{\lambda t}{t} = \lambda$ and $\lim_{t\to\infty} Var(\frac{N(t)}{t}) = \lambda$.

In otherwords $\frac{N(t)}{t}$converges to λ as $t \to \infty$. Because of this λ is called the arrival rate of the poisson process.

3. Under very general assumptions the sum of a large number of renewal

process behaves like a Poisson process.

4. Let $N_i = [N_i(t) : t \geq (0)]$; $i = 1, 2, ...n$ be n independent Poisson process with renewal rates $\lambda_1, \lambda_2, ..., \lambda_n$ Then the superposition of $N_i = 1, 2, ...n$ is also a Poisson variate with arrival rates $\lambda_1, \lambda_2, ...\lambda_n$.

5. Poisson process can be identified as a renewal process with exponentially distributed intervals.

6. The Poisson process is a Markov process.

7. The sum of two Independent Poisson process is a Poisson process.

8. The difference of two Independent Poisson process is not a Poisson process.

9. The inter arrival time of a Poisson process with parameter λ has an exponential distribution with mean $\frac{1}{\lambda}$.

10. The probability law of the Poisson process $\{X(t)\}$ is the same as that of a Poisson distribute with parameter λt.

2.4 GAUSSIAN PROCESS

In probability theory and statistics, a Gaussian process is a stochastic process (a collection of random variables indexed by time or space), such that every finite collection of those random variables has a multivariate normal distribution. The distribution of a Gaussian process is the joint distribution of all those (infinitely many) random variables, and as such, it is a distribution over functions with a continuous domain, e.g. time or space.
Gaussian Processes (GP) are a nonparametric supervised learning method used to solve regression and probabilistic classification problems.

A Gaussian random variable $X \sim N(\mu, \Sigma)$, where μ is the mean and Σ is the covariance matrix has the following probability density function:

$$P(x;\mu,\Sigma) = \frac{1}{(2\pi)^{d/2}|\Sigma|} e^{\frac{-1}{2}((x-\mu)\Sigma^{-1}(x-\mu)}$$

where $|\Sigma|$ is the determinant of Σ

Gaussian processes are useful in statistical modelling, benefiting from properties inherited from the normal distribution. For example, if a random process is modelled as a Gaussian process, the distributions of various derived quantities can be obtained explicitly. Such quantities include the average value of the process over a range of times and the error in estimating the average using sample values at a small set of times. While exact models often scale poorly as the amount of data increases, multiple approximation methods have been developed which often retain good accuracy while drastically reducing computation time.

Gaussian Process Regression has the following properties:

- GPs are an elegant and powerful ML method.
- We get a measure of (un)certainty for the predictions for free.
- GPs work very well for regression problems with small training data set sizes.
- GPs are a little bit more involved for classification (non-Gaussian likelihood).
- We can model non-Gaussian likelihoods in regression and do approximate inference for e.g., count data (Poisson distribution)
- GP implementations: GPyTorch, GPML (MATLAB), GPys, pyGPs, and scikit-learn (Python)

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2.5 BROWNIAN PROCESS

Brownian process is another widely-used random process. It has been used in engineering, finance, and physical sciences. It is a Gaussian random process and it has been used to model motion of particles suspended in a fluid, percentage changes in the stock prices, integrated white noise, etc.

Examples of brownian motion include:

- Movement of pollen grains in water.
- Movement of dust particles in air.
- Diffusion of pollutants in air.
- Movement of holes of electrical charge in semiconductors.

A Brownian motion (or Wiener Process) is a stochastic process$[X(t), t \geq 0]$ with the following properties:

(i) Every increment $X(t)-X(s)$ during (s,t) is normal with mean $c(t-s)$ and variance $\sigma^2(t-s)$

(ii) For all $0=t_0 < t_1 < ...t_n < \infty$, and n, the increments $X(t_1) - X(t_0), ..., X(t_n) - X(t_{n-1})$ are independent.

When $c=0$, $\sigma^2 = 1$, we have the *standard Brownian motion* denoted by W(t), where we may take W(0)=0.

ELEMENTARY PROPERTIES

The following elementary properties of W(t) follow from the definition and the properties of the normal distribution.

Property 1: The Weiner process W(t) is a Markov process with transition distribution function

$$\begin{aligned} F(x,\, t; x_0,\, s) =& P[W(t) \leq x | W(s) = x_0] \\ =& \int_{-\infty}^{x} \frac{1}{\sqrt{4\pi(t-s)}} \exp\left[-\frac{u-x_0)^2}{2(t-s)}\right] du, \\ =& F(x-x_0, t-s) \end{aligned}$$

Property 2: There is a consistent system of probability distributions of $[W(t), t \geq 0]$

Property 3: For the standard Wiener process $W(t)$, with $W(t_1) = a, W(t_2) = b, t_1 < t < t_2$, the conditional distribution of $W(t)$ is normal with

$$mean = a + \frac{b-a}{t_2 - t_1}(t - t), variance = \frac{(t_2 - t)(t - t_1)}{t_2 - t_1}$$

The Property 3 can be established as follows

$$Y = W(t) - W(t_1), Z = W(t_2) - W(t)$$

are independent normal with means 0 and variances $(t - t_1)$ and $(t_2 - t)$ respectively.

Property 4: (Symmetry Properties). If $[W(t), t \geq 0)]$ is a standard Wiener process, so also are the following processes:

(i)$W_1(t) = c\ W(t/c^2), t > 0$ (scale symmetry),

(ii)$W_2(t) = W(t + h) - W(h) h \geq 0, t \geq 0$ (translation of increments).

(iii) $W_3(t) = tW(t^{-1})(t > 0)$(inversion),

(iv)$W_4(t) = -W(t).(t \geq 0)$ (reflection).

2.6 BIRTH AND DEATH PROCESS

In stochastic processes, a pure birth process is a specific type of continuous-time Markov process where events occur at a constant rate. In a pure birth process, the state of the system represents the number of entities present at any given time, and entities are born (or added) to the system at a rate

Property 2: There is a consistent system of probability distributions of $[W(t), t \geq 0]$

Property 3: For the standard Wiener process $W(t)$, with $W(t_1) = a, W(t_2) = b, t_1 < t < t_2$, the conditional distribution of $W(t)$ is normal with

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Key characteristics of a pure birth process in the context of stochastic processes include:

- Continuous Time: Events occur continuously over time rather than at discrete time point
- Markov Property: The future behavior of the process depends only on its current state and not on its past states, given the current state
- Birth Rate: Entities are born into the system at a constant rate, with the rate typically being proportional to the number of entities already present in the system
- No Death: In a pure birth process, entities are only added to the system; there are no mechanisms for removal or death
- Memorylessness: The inter-arrival times between successive events (births) are exponentially distributed, implying that the process has no memory of how much time has passed since the last event.
- State Space: The state space of the process is typically the set of nonnegative integers (0, 1, 2, ...) representing the number of entities present in the system.

Pure birth processes find applications in various fields, including population dynamics, epidemiology, and branching processes. They provide a mathematical framework for modeling systems characterized by continuous growth without any limiting factors such as competition or death.

We have studied in the Poisson proces that in an intervel of infinitesimal length h, the probability of exactly one occurrence is $\lambda h + O(h)$ and that of more than one occurrence is of $O(h)$. Here $O(h)$ is used as a symbol to denote a function of h which tends to 0 more rapidly than h (i.e) As $h \to 0$, $\frac{O(h)}{h} \to 0$ Therefore in the intervel $(t, t + h)$

$$P(N(h) = 1) = \lambda h + O(h)$$

$$\sum_{k=2}^{\infty} P(N(h) = k) = O(h)$$

But we have $\sum_{k=0}^{\infty} P(N(h) = k) = 1$

$$\Rightarrow P(N(h) = 0) = 1 - \lambda h + O(h)$$

In the classical Poisson process the conditional probabilities are constant. Here the probability that k events occur between t and $t + h$ given that n events occured in $(0, t)$ is given by

$$\begin{aligned} P(N(h) = k/N(t) = n) &= \lambda h + O(h), & k = 1 \\ &= O(h), & k \geq 2 \\ &= 1 - \lambda h + O(h), & k = 0 \end{aligned}$$

which is independent of t as well as n. We can generalise this process by considering that λ is not a constant but is a function of n or t or both. The resulting process will sill be Markovian in character.

Case (i): First let us consider the case when λ to be a function of n, the population size at the instant.

$$\text{Now,} \quad \begin{aligned} P_n(h) = P(N(h) = k/N(t) = n) &= \lambda_n h + O(h), & k = 1 \\ &= O(h), & k \geq 2 \\ &= 1 - \lambda_n h + O(h), & k = 0 \end{aligned}$$

Proceeding as in the Poisson distribution we have

$$\begin{aligned} & P_n(t + h) = P_n(t)\,(1 - \lambda_n h) + P_{n-1}(t)\lambda_{n-1}(h) + O(h) n \geq 1 \\ \Rightarrow\ & P_n'(t) = -\lambda_n P_n(t) + \lambda_{n-1} P_{n-1}(t), n \geq 1 \\ & P_o'(t) = -\lambda_n P_n(t) \end{aligned}$$

For given initial conditions explicit expressions for $P_n(t)$ can be obtained from the above equations. This process is called a pure birth process. Now the process corresponding to $\lambda_n = n\lambda$ is called the Yule-Furry process.

2.7 BRANCHING PROCESS

A branching process is a stochastic process that models the evolution of a population over discrete time steps, where individuals in the population can reproduce and produce offspring according to certain probabilistic rules. In a branching process, the population at each generation is composed of individuals, and each individual can independently give rise to a random number of offspring. The offspring distribution typically follows a probability distribution, which may vary between individuals.

There are different types of branching processes depending on the assumptions made about the population's characteristics. The most common branching process is the Galton-Watson process, which is a discrete-time model that assumes that the number of offspring produced by each individual is a fixed, non-negative integer.

The branching process include the mean and variance of the offspring distribution, which determine the growth or decline of the population over time. If the mean number of offspring per individual is greater than 1, the population is expected to grow exponentially. Conversely, if the mean is less than 1, the population is expected to decline and eventually go extinct.

The study of branching processes often involves techniques from probability theory, including generating functions, Markov chains, and martingale theory. Branching processes provide a powerful framework for understanding the dynamics of populations and systems that involve random reproduction and growth.

The most common formulation of a branching process is that of the Galton–Watson process. Let Z_n denote the state in period n and let $X_{n,i}$ be a random variable denoting the number of direct successors of member i in period n, where $X_{n,i}$ are independent and identically distributed random variables over all $n \in (0, 1, 2, ...)$ and $i \in (1, ..., Z_n)$. Then the recurrence equation is

$$Z_{n+1} = \sum_{i=1}^{Z_n} X_{n,i}$$

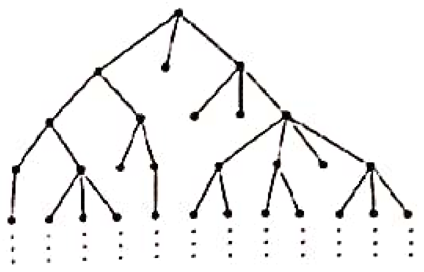

Figure 1: Branching process figure construction

with $Z_0 = 1$.
Alternatively, the branching process can be formulated as a random walk. Let S_i denote the state in period i, and let X_i be a random variable that is iid over all i. Then the recurrence equation is

$$S_{i+1} = S_i + X_{i+1} - 1 = \sum_{j=1}^{i+1} X_j - i$$

with $S_0 = 1$.For this formulation, Let Si represent the number of revealed but unvisited nodes in period i, and let Xi represent the number of new nodes that are revealed when node i is visited. Then in each period, the number of revealed but unvisited nodes equals the number of such nodes in the previous period, plus the new nodes that are revealed when visiting a node, minus the node that is visited. The process ends once all revealed nodes have been visited.

Chapter-3

APPLICATIONS OF STOCHASTIC PROCESS

Stochastic process is widely used as a mathematical model of systems and phenomena that appear to vary in a random manner. As a classic technique from statistics, stochastic processes are widely used in a variety of areas including bioinformatics, neuroscience, image processing, financial markets, etc. Here it is discussed about how stochastic process is related to machine learning and what are its major application areas.

Below are some general and popular applications which involve the stochastic processes:-

- Stochastic models are used in financial markets to reflect the seemingly random behaviour of assets such as stocks, commodities, relative currency values (i.e., the price of one currency relative to another, such as the price of the US Dollar relative to the price of the Euro), and interest rates.

- Manufacturing procedures are thought to be stochastic. This assumption holds true for both batch and continuous manufacturing processes. A process control chart depicts a particular process control parameter across time and is used to record testing and monitoring of the process.

- The marketing and shifting movement of audience tastes and preferences, as well as the solicitation and scientific appeal of the certain film and television debuts (i.e., opening weekends, word-of-mouth, top-of-mind knowledge among surveyed groups, star name recognition, and other elements of social media outreach and advertising), are all influenced in part by stochastic modelling.

- Stanislaw Ulam and Nicholas Metropolis popularized the Monte Carlo approach, which is a stochastic method. The use of randomness and the repetitive nature of the procedure is reminiscent of casino activities. Simulation and statistical sampling methods were typically used to test a previously understood deterministic problem, rather than the other way around. Though historical examples of an "inverted" technique exist, they were not regarded as a generic strategy until the Monte Carlo method gained popularity.

- Probabilistic Models: Stochastic processes are used to build probabilistic models that can capture the uncertainty in data and make predictions based on probabilistic reasoning. Bayesian inference, hidden Markov models, and Gaussian processes are examples of probabilistic models that rely on stochastic processes.

- Time Series Analysis: Stochastic processes are widely used in time series analysis, where data points are collected sequentially over time. Models such as autoregressive integrated moving average (ARIMA) and stochastic volatility models use stochastic processes to model the underlying dynamics of time series data.

- Generative Models: Stochastic processes are fundamental to generative models, which are used to generate new data samples from a given distribution. Variational autoencoders (VAEs) and generative adversarial networks (GANs) are examples of generative models that rely on stochastic processes.

- Optimization: Stochastic optimization algorithms, such as stochastic gradient descent (SGD), use stochastic processes to optimize complex objective functions that involve randomness or noise. These algorithms are commonly used in training machine learning models.

- Natural Language Processing: Stochastic processes are used in various natural language processing tasks, such as language modeling, text

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- Natural Language Processing: Stochastic processes are used in various natural language processing tasks, such as language modeling, text generation, and machine translation. Probabilistic models based on stochastic processes help capture the uncertainty and variability in language data.

Stochastic processes find applications in a wide array of fields due to their ability to model randomness and uncertainty in various systems.Stochastic processes have wide relevance in mathematics both for theoretical aspects and for their numerous real-world applications in various domains. They represent a very active research field which is attracting the growing interest of scientists from a range of disciplines.

In particular, the focus here is on applications of stochastic processes as models of dynamic phenomena in research areas certain to be of interest, such as economics, statistical physics, biology, theoretical neurobiology, and reliability theory. Various contributions dealing with theoretical issues on stochastic processes are also included.Stochastic processes are the key tools for modeling and reasoning in many physical and engineering systems. The stochastic process is a probability model that represents the possible sample paths as a collection of time-ordered random variables. As a mathematical model, it is widely used to study phenomena and systems that seem to vary randomly. Stochastic processes are a classic technique from statistics that are widely used in a wide range of fields including bioinformatics, neuroscience, image processing, the financial markets, and others.In machine learning, stochastic gradient descent and stochastic gradient boosting are the two most common algorithms.

Several machine learning methods and models use stochasticity to explain their results. The reason is that many optimizations and learning algorithms work in stochastic domains, and some algorithms depend on randomness or probabilistic decisions. In this section, we will examine the sources of uncertainty and the nature of stochastic algorithms in machine learning. In the financial markets, stochastic models are used to reflect seemingly random patterns of asset prices such as stocks, commodities, relative currency values (e.g., the price of the US Dollar relative to the price of the Euro), and interest rates. Manufacturing processes are considered stochastic. This assumption applies to both batch and continuous manufacturing. An example of a process control chart is a chart that records the performance of a particular process control parameter over time.

Stochastic modeling informs some aspects of marketing, shifting audience preferences, crowdsourcing, and the scientific appeal of certain film and television debuts (i.e., opening weekends, word-of-mouth, top-of-mind knowledge, star recognition, and other social media outreach and advertising strategies).

Although stochastic process theory and its applications have made great progress in recent years, there are still a lot of new and challenging problems existing in the areas of theory, analysis, and application, which cover the fields of stochastic control, Markov chains, renewal process, actuarial science, and so on. These problems merit further study by using more advanced theories and tools.

Overall, stochastic processes play a crucial role in machine learning by providing a mathematical framework to model uncertainty, variability, and randomness in data and algorithms. Their applications span a wide range of domains, including finance, manufacturing, marketing, simulation, and optimization.

Chapter-4

QUEUEING THEORY

Introduction

Queueing theory is the mathematical study of waiting lines, or queues.A queueing model is constructed so that queue lengths and waiting time can be predicted. Queueing theory is generally considered a branch of operations research because the results are often used when making business decisions about the resources needed to provide a service.

Queueing theory dates back to A. K. Erlang's (1878-1929) fundamental work on the study of congestion in telephone traffic, and since then it has been applied to a wide variety of applications such as inventory control, road traffic congestion, aviation traffic control, machine interference problem, biology, astronomy, nuclear cascade theory and, of course, voice and data communication networks. Simple queues collectively form a chain of queues, where queues, in turn, feed other queues, and this process can go on for several layers forming complex networks of queues. The mathematical characterization and study of these phenomena constitute queueing theory.

A queue, or a waiting line, is formed by arriving customers/jobs requiring service from a service station. If service is not immediately available, the arriving units may join the queue and wait for service and leave the system after being served, or may leave sooner without being served for various reasons. In the meantime, other units may arrive for service. The source from which the arriving units come may be finite or infinite. An arrival may consist of a single unit or in bulk. The service system may have either a limited or unlimited capacity for holding units, and depending on that, an arriving unit may join or leave the system. Service may be rendered either singly or in bulk. The basic features of a queue are:
(i) the input process,
(ii) the service mechanism,
(iii) the queue discipline and
(iv) the server's capacity.

Queuing Models in Operating System

In general, there is no fixed set of processes that run on systems; thus, measuring the exact processing requirements of processes is impossible. We can, however, measure the distributions of CPU bursts and I/O bursts over the course of a process and derive a mathematical formula that identifies the probability of a specific CPU burst. The arrival rate of processes in the system can be approximated in the same way. The development of queuing theory, a branch of mathematics, resulted from the use of mathematical models for evaluating the performance of various systems. The fundamental model of queuing theories is the same as the model of a computer system. Each computer system is represented as a collection of servers such as CPUs and I/O devices, each with its own queue.

Components of Queuing System

A queuing system typically includes the following elements:

- Arrival process: The arrival process describes how customers enter the system.

- Server: The server is the person who provides the service to the customers.

- Queue: Customers who are waiting for service are held in a queue

- Service discipline: The order in which customers are served is determined by service discipline.

- Service time distribution: The amount of time required to serve a customer is described as service time distribution.

- Departure process: The departure process describes how customers exit the system once they have been served.

- System performance measures: System performance measures are used to analyze and evaluate the system's performance. Examples include the average wait time, the number of customers in the system, and the server's utilization.

Optional extras include multiple servers or channels, priority service, and feedback or renege mechanism.

Number of Servers

The number of servers in a queuing system can vary depending on the application and the level of service desired. In some cases, a single server may suffice, whereas, in others, multiple servers may be required to meet demand.

- Single-server queuing systems: These are the most fundamental type of queuing systems, and they are frequently used in simple applications such as retail stores or fast-food restaurants. Customers arrive and queue to be served by a single server in these systems.

- Multi-server queuing systems: Multi-server queuing systems on the other hand, are used in more complex applications where demand is high and more than one server is required to handle the workload. A call center with multiple agents to handle incoming calls is an example of this type of system. Customers are usually directed to an available server in a multi-server system, and the service time distribution is assumed to be the same across all servers.

Various methods, such as queuing analysis, simulation, and optimization techniques, can be used to determine the number of servers in a queuing system. The goal is typically to find the optimal number of servers that minimizes system costs (e.g., staff wages) while providing an acceptable level of service.

Measures of Performance for Queuing Systems

Performance measures for queuing systems are used to assess how well the system is performing and to identify areas for improvement. Some common performance indicators for queuing systems are:

- Utilization: The percentage of time spent by the server serving customers. A high utilization rate indicates that the server is being used effectively, whereas a low utilization rate indicates that the server is being underutilized

- Average waiting time: The amount of time customers spend waiting in line to be served. A long waiting time may indicate a system bottleneck, whereas a short waiting time indicates that the system is running efficiently.

- An average number of customers in the system: The average number of customers in the system, including those being served as well as those waiting in line. A high number of customers in the system may indicate that there is a high demand for service, whereas a low number indicates that the system is running efficiently.

- An average number of customers in line: The average number of customers in line to be served. A large number of customers in the queue may indicate that the system is unable to meet the demand for service, whereas a small number indicates that the system is operating efficiently.

- Throughput: The rate at which the system serves customers. A high throughput indicates that the system is running efficiently, whereas a low throughput may indicate that the system has a bottleneck.

Notation for Queues

Kendall's notation and A/S/n notation are two popular notations for describing queues.

- Kendall's notation: This describes a queue by using a set of symbols to represent the queue's various characteristics. It is represented by a three-letter notation, with each letter representing a different aspect of the queue. The first letter denotes the arrival process, the second the service process, and the third the number of servers.

- A/S/n notation: In this notation A represents the probability distribution of the interarrival time, S represents the service time distribution, and n represents the number of servers.

Queue Discipline

The order in which customers are served in a queuing system is referred to as queue discipline. In practice, there are several queue disciplines that are used, including:

- First-In-First-Out (FIFO): Customers are served in the order in which they arrive (first-in, first-out). This is the most commonly used queue discipline in retail stores, fast-food restaurants, and other similar establishments.

- Last-In-First-Out (LIFO): Customers are served in reverse order of arrival (last-in-first-out, or LIFO). This discipline is less commonly used than FIFO, but it can be found in some applications, such as a stack of plates in a cafeteria.

- Priority: Customers are served in accordance with their priority level. Customers with the highest priority are served first, followed by customers with lower priority. This discipline is used in situations where certain customers, such as in an emergency room or a customer service call center, must be served before others.

- Random: Customers are served at random. Shortest Job first (SJF): Customers are served based on the time required to complete their service, with the shortest jobs served first.

- Processor sharing: Processor sharing means that all customer requests are treated equally and receive an equal share of the server's time.

Queuing Models

Below are the four queuing models that will be discussed here:

1. [M/M/1]: {//FCFS} Queue System

M/M/1 denotes a queueing system with one server and a Poisson distribution for customer interarrival times and service times. The notation /FCFS indicates that a first-come-first-served (FCFS) service discipline is being used, which means that customers are served in the order in which they arrive. This type of queue is also known as an M/M/1/FCFS queue or an M/M/1/FIFO (first-in-first-out) queue. It is one of the most basic and widely studied queueing models in queuing theory, and it is frequently used as a starting point for understanding the performance of more complex queueing systems. To analyze and evaluate an M/M/1 queue, several performance measures are commonly used. Among the most important measures are:

1. The average number of customers in the system
2. The average waiting time in the queue.
3. The system utilization.
4. The probability of a customer finding the server busy.

These metrics can be computed using a variety of analytical techniques, including Queueing formulae, Markov Chain analysis, and even numerical methods. In the case of the M/M/1 queueing model, closed-form solutions for these measures are available, making the analysis relatively simple.

2. [M/M/1]: {N//FCFS} System (Limited queue length system)

It is a single-server queueing system with a Poisson arrival process and an exponential service time distribution. N denotes a limited queue length, implying that the queue can only hold a certain number of customers. The notation N/FCFS indicates that the service discipline is first-come-first-served (FCFS), which means that customers are served in the order in which they arrive, but also that when the queue is full, new arriving customers are blocked or rejected, a practice is known as Balking and Reneging. An M/M/1/FCFS/N or M/M/1/FIFO/N queue is another name for this type of queuing system. It is a variant of the basic M/M/1 queue with a limited buffer capacity, which means that the number of customers in the system is limited by N. When the buffer is full, additional customers may be blocked or rejected, complicating the analysis and necessitating the modification of some performance measures to include the blocked/rejected customers. Some of the most important performance measures that can be calculated. similar to an M/M/1 system, are:

1. The average number of customers in the system (including those blocked or rejected).
2. The average waiting time in the queue.
3. The system utilization.
4. The probability of a customer finding the server busy.
5. The probability of customers being blocked/reneged.

3. M/D/1 Queue

The M/D/1 queue is a queuing system in which customer arrival times follow a Poisson distribution (M), service times are deterministic (D) and have a constant value, and the system has one server. This is also known as an M/D/1/FCFS or M/D/1/FIFO queue, where FCFS or FIFO denotes the first-come-first-served service discipline. This type of queuing system is useful for simulating situations where customer service times are known in advance and are consistent, such as a carwash service. The queue will be stable if the arrival rate is less than the service rate, otherwise, it will be unstable due to the deterministic service time. The performance measures for this system are similar to those for the M/M/1 queue, but because the service time is deterministic, closed-form solutions for these measures are frequently easier to obtain. Among the most important measures are:

1. The average number of customers in the system.

2. The average waiting time in the queue.

3. The system utilization.

4. The probability of a customer finding the server busy.

Furthermore, the queue length is predictable in this case, which means that given a specific arrival rate and service time, the number of customers in the system will always be the same, rather than being dependent on the randomness of the service time as it is in M/M/1.

4. M/M/c Queue

The M/M/c queue is a queuing system in which customer arrival times follow a Poisson distribution (M), service times are also exponentially distributed (M), and the system has c servers. This is also known as an M/M/c/FCFS or M/M/c/FIFO queue, where FCFS or FIFO denotes the first-come-first-served service discipline. It is also known as the Erlang-c queue. This type of queuing system is useful for simulating situations in which multiple servers provide service. This queuing system allows customers to be served concurrently by the c servers, increasing system capacity and decreasing average customer wait time. This system's performance metrics are similar to those of the M/M/1 queue. However, some measures are more difficult to calculate because the number of servers c influences the queue's behavior. The most important measures are as follows:

1. The average number of customers in the system.

2. The average waiting time in the queue

3. The system utilization.

4. The probability of a customer finding the server busy

5. The probability of customers waiting in the queue.

The probability of customers being blocked. The M/M/c queue is a more complex model than the M/M/1 queue, and the performance measures are calculated using various approximate and numerical methods. Furthermore, when the number of servers is large, as in many industrial and telecommunications systems, the system can be approximated as an M/M/c queue.

Applications of Queuing Theory

- **Finance:**

  By applying queuing theory, a business can develop more efficient systems, processes, pricing mechanisms, staffing solutions, and arrival management strategies to reduce customer wait times and increase the number of customers that can be served.Therefore, businesses use information gleaned from queuing theory in order to set up their operational functions so as to strike a balance between the cost of servicing customers and the inconvenience to customers caused by having to wait in line.

- **Telecommunication:**

  The fundamental unit of telecommunications traffic in voice systems is called the Erlang. Queuing theory as an operations management technique is commonly used to determine and streamline staffing needs, scheduling, and inventory in order to improve overall customer service.

- **Traffic control:**

  The application of the queuing theory is exploited to minimized the traffic congestion at a particular time. By this work we find out different steps to avoid the congestion.

  - The traffic can be reduced by increasing road capacity
  - We can provide separate lane for specific user group.

- **Health services:**

  In health care, queuing models are generally based on three factors and the variation within. Those factors are patient arrival rate, server rate (service time for exam, treatment, etc.) and the number of servers (clinical and nonclinical staff) available.

- **Computer Networks:**

  A Queue Data Structure is a fundamental concept in computer science used for storing and managing data in a specific order. It follows the principle of "First in, First out" (FIFO), where the first element added to the queue is the first one to be removed.

CONCLUSION

This Project contained four chapters.The Preliminary which included definition of Probability, Probability space,Outcome,Mean, Variance, Random Variable,Binary random variables,Normal distribution, etc.

First chapter dealt with the introduction to stochastic process.A stochastic process is a mathematical model that describes the evolution of a system over time in a probabilistic manner. It consists of a collection of random variables indexed by time, space, or other parameters. These random variables represent the uncertain or random aspects of the system's behavior, and the stochastic process captures how they change and interact with each other over time.An examples that includes the growth of a bacterial population, an electrical current fluctuating due to thermal noise, or the movement of a gas molecules.

Stochastic processes are widely used in various fields such as finance, engineering, biology, physics, and more to model and analyze complex systems that involve randomness or uncertainty. They provide a powerful framework for understanding the underlying patterns, trends, and uncertainties in data and can be used to make predictions, simulate scenarios, and optimize decision-making processes.

There are different types of stochastic processes, including Markov processes, Brownian motion, Poisson processes, and more, each with its own characteristics and applications. By studying the properties and behavior of stochastic processes, researchers and practitioners can gain valuable insights into the dynamics of random phenomena and make informed decisions in the face of uncertainty.

In this Second chapter, type of stochastic process including the Bernoulli process, Poisson random process, Brownian process,Birth and Death process. The Bernoulli process,which is a fundamental concept in probability and statistics.We have discussed the properties of the Bernoulli process, including the expected value, variance, and relationships between consecutive variables. how the partial sums of a Bernoulli process form a sequence and how they can be viewed as a Markov process.The Bernoulli process also including Binomial random process, Non-homogeneous Bernoulli process,Random walk.The Poisson random process which included some basic definition and properties of a fundamental stochastic process that models random arrivals in continuous time, characterized by interarrival times following a geometric distribution and defined by postulates. The Brownian motion, or Wiener process, is a stochastic process characterized by normal increments with specific mean and variance properties. Its elementary properties include the Markov property and the independence of normal variates in the sum of increments, leading to normality with zero mean and variance equal to the sum of individual variances.Birth and Death processes, exemplified by the Yule-Furry process, offer a simplified yet insightful framework for studying population dynamics and other stochastic phenomena. While these models have limitations, they provide a foundational understanding of how populations evolve over time and can be extended to more realistic scenarios for further analysis.The Simple Birth Process for unicellular organisms assumes that all individuals can reproduce, leading to exponential population growth over time. This model provides a basic framework for studying population dynamics and growth patterns in simple organisms.

Third Chapter including Stochastic processes play a crucial role in various fields, including bioinformatics, neuroscience,image processing, financial markets,etc. where they are utilized as mathematical models to capture random variations in systems and phenomena. The application of stochastic processes is the analysis and prediction of complex data patterns that exhibit random behavior.

Fourth chapter,Queueing theory including Queuing Models in Operating System,Components of Queuing System,Number of Servers,Measures of Performance for Queuing Systems,Notation for Queues,Queue Discipline,Queuing Models and Applications of Queuing Theory.There are four types of Queuing Models :[M/M/1]: //FCFS Queue System,[M/M/1]: N//FCFS System (Limited queue length system),M/D/1 Queue and M/M/c Queue.

Queueing theory is a powerful mathematical tool that allows us to study and optimize the behavior of waiting lines in various systems. By analyzing factors such as arrival rates, service rates, queue lengths, and customer behavior, we can make informed decisions to improve efficiency, reduce waiting times, and enhance customer satisfaction. Queueing theory has applications in a wide range of industries, from healthcare and transportation to telecommunications and manufacturing, making it a valuable tool for improving operational performance. By understanding and applying Queueing theory principles, organizations can streamline processes, allocate resources effectively, and ultimately provide better service to their customers.

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