MATHS PROJECT

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History of Calculus

History of Calculus is part of the history of mathematics focused on limits, functions, derivatives, integrals , and infinite series. The subject, known historically as infinitesimal calculus , constitutes a major part of modern mathematics education. It has two major branches, differential calculus and integral calculus, which are related by the fundamental theorem of calculus . Calculus is the study of change, in the same way that geometry is the study of shape and algebra is the study of operations and their application to solving equations.

The Indian mathematician-astronomer Aryabhata in 499 used a notion of infinitesimals and expressed an astronomical problem in the form of a basic differential equation. Manjula, in the 10th century, elaborated on this differential equation in a commentary. This equation eventually led Bhāskara II in the 12th century to develop the concept of a derivative representing infinitesimal change, and he described an early form of "Rolle's theorem."

Newton and Leibniz are usually credited with the invention of modern infinitesimal calculus in the late 17th century. Their most important contributions were the development of the fundamental theorem of calculus. Before Newton and Leibniz, the word “calculus” was a general term used to refer to any body of mathematics, but in the following years, "calculus" became a popular term for a field of mathematics based upon their insights.

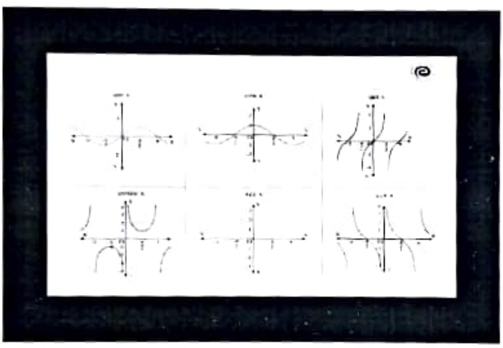
Basic Formulas Of Differentiation

Derivative Formulas of Elementary Functions

- d/dx .x^n = n. x^{n-1}
- d/dx .k = 0, where k is a constant
- d/dx .e^x = e^x
- d/dx .a^x = a^x. log.a , where a > 0, a ≠ 1
- d/dx. logx = 1/x, x > 0
- d/dx. loga e = 1/x loga e
- d/dx.√x =1/(2 √x)

Derivatives of trigonometric functions

- $(d/dx) \sin x = \cos x$
- $(d/dx) \cos x = -\sin x$
- $(d/dx) \tan x = \sec^2 x$
- $(d/dx) \operatorname{cosec} x = -\operatorname{cosec} x \cot x$
- $(d/dx) \sec x = \sec x \tan x$
- $(d/dx) \cot x = -\operatorname{cosec}^2 x$



Examples:

1) $y = 12x^2 - 16x + 4$

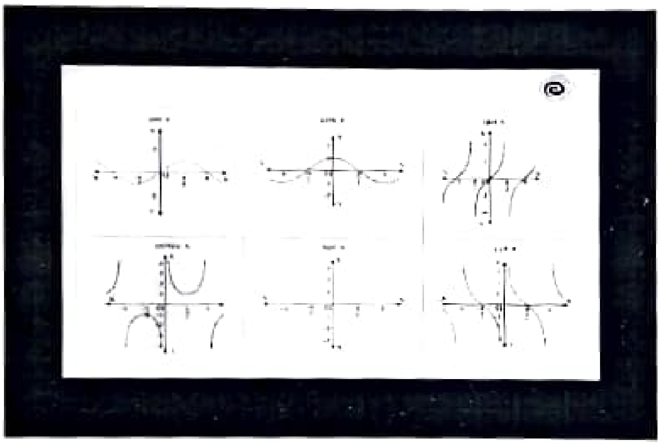
Answer: $y'' = 24$.

2) $y = x^3 - 4x^2 + 1/x;\ x \neq 0$

Answer: $6x - 8 + 2/x^3$.

Derivatives of trigonometric functions

- (d/dx) sin x = cos x
- (d/dx) cos x = -sin x
- (d/dx) tan x = $\sec^2$ x
- (d/dx) cosec x = -cosec x cot x
- (d/dx) sec x = sec x tan x
- (d/dx) cot x = -cosec^2 x



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Answer: $6x - 8 + 2/x^3$.

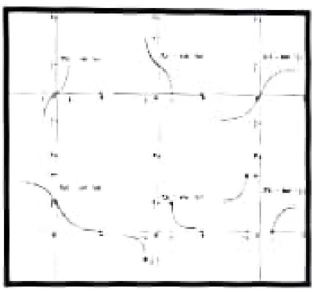
3) $y = \cos 3x$

Answer: $y'' = -9\cos 3x$.

4) $y = \sin^3 x$

Answer: $y'' = 9\sin^3 x - 6\sin x$.

Derivatives of inverse trigonometric functions.



$d/dx\,(\sin^{-1}x) = 1/\sqrt{1-x^2}$

$d/dx(\cos^{-1}x) = -1/\sqrt{1-x^2}$

$d/dx(\tan^{-1}x) = 1/(1+x^2)$

$d/dx(\csc^{-1}x) = -1/(|x|\sqrt{x^2-1})$

$d/dx(\sec^{-1}x) = 1/(|x|\sqrt{x^2-1})$

$d/dx(\cot^{-1}x) = -1/(1+x^2)$

Examples

Problem 1: $y = \tan^{-1}(1/x)$

$y' = (\tan^{-1}x)'$

$= \{1/1 + (1/x^2)\} . (-1/x^2)$

$= - x^2 / (x^2 + 1) . x^2$

Problem 2: $y = \sin^{-1}(1 - x)$

$y' = (\sin^{-1}(1 - x))'$

$= 1 / 1 - (x - 1)^2$

$= 1 / \sqrt{(1 - (x^2 - 2x + 1))}$

$= 1 / \sqrt{(2x - x^2)}$

Problem 3: $y = (1/a) \tan^{-1}(x/a)$

$y' = ((1 / a) \tan^{-1}(x / a))'$

$= (1 / a) \{1 / (1 + (x / a))\} . (x / a)'$

$= 1 / a . \{1 / (1+ (x^2 / a^2))\} . (1 / a)$

$= 1 / a^2 . \{a / (a^2 + x2)\}$

$= a / a^2 + x^2$

Problem 4: $y = \cot^{-1}(1/x^2)$

$y' = (\cot^{-1}(1 / x^2))'$

$= \{- 1 / (1 + (1 / x^2))^2 \} . (1 / x^2)'$

$= \{- 1 / (1 + (1 / x^4)) . (-2x^{-3})$

$= 2x^4 / (x^4 + 1)x^3$

$= 2x / (1 + x^4)$

Types of Differentiation

Derivatives Types

Derivatives can be classified into different types based on their order such as first and second order derivatives. These can be defined as given below.

First-Order Derivative

The first order derivatives tell about the direction of the function whether the function is increasing or decreasing. The first derivative math or first-order derivative can be interpreted as an instantaneous rate of change. It can also be predicted from the slope of the tangent line.

Second-Order Derivative

The second-order derivatives are used to get an idea of the shape of the graph for the given function. The functions can be classified in terms of concavity. The concavity of the given graph function is classified into two types namely:

- **Concave Up**
- **Concave Down**

Product Rule

Product rule in calculus is a method to find the derivative or differentiation of a function given in the form of the product of two differentiable functions. That means, we can apply the product rule, or the Leibniz rule, to find the derivative of a function of the form given as: $f(x)\cdot g(x)$, such that both $f(x)$ and $g(x)$ are differentiable. The product rule follows the concept of limits and derivatives in differentiation directly.. That means if we are given a function of the form: $f(x)\cdot g(x)$, we can find the derivative of this function using the product rule derivative as,

$f(x)\cdot g(x) = [g(x) \times f'(x) + f(x) \times g'(x)]$

Example: Find $f'(x)$ for the following function $f(x)$ using the product rule: $f(x) = x\cdot\log x$.

Here, $f(x) = x\cdot\log x$

$u(x) = x$

$v(x) = \log x$

$\Rightarrow u'(x) = 1$

$\Rightarrow v'(x) = 1/x$

$\Rightarrow f'(x) = [v(x)u'(x) + u(x)v'(x)]$

$\Rightarrow f'(x) = [\log x\cdot 1 + x\cdot(1/x)]$

$\Rightarrow f'(x) = \log x + 1$

i) Find the derivative of $x\cdot \cos(x)$ using the product rule formula.

Let $f(x) = \cos x$ and $g(x) = x$.

$\Rightarrow f'(x) = -\sin x$

$\Rightarrow g'(x) = 1$

$\Rightarrow [f(x)g(x)]' = [g(x)f'(x) + f(x)g'(x)]$

$\Rightarrow [f(x)g(x)]' = [(x \cdot (-\sin x) + \cos x \cdot (1)]$

$\Rightarrow [f(x)g(x)]' = - x \sin x + \cos x$

ii) Differentiate x2 log x using the product rule formula.

Let f(x) = log x and g(x) = x2.

$\Rightarrow f'(x) = (1/x)$

$\Rightarrow g'(x) = 2x$

$\Rightarrow [f(x)g(x)]' = [g(x)f'(x) + f(x)g'(x)]$

$\Rightarrow [f(x)g(x)]' = [(x2 \cdot (1/x) + \log x \cdot (2x)]$

$\Rightarrow [f(x)g(x)]' = x + 2x \log x$

Chain Rule

The chain rule is used to find the derivatives of composite functions like (x2 + 1)3, (sin 2x), (ln 5x), e2x, and so on. If $y = f(g(x))$, then $y' = f'(g(x)).\ g'(x)$. The chain rule states that the instantaneous rate of change of f relative to g relative to x helps us calculate the instantaneous rate of change of f relative to x.

i) Find the derivative of $y= \ln \sqrt{x}$ using the chain rule

$y = \ln \sqrt{x}$.

$f(x) = y$ is a composition of the functions $\ln(x)$ and $\sqrt{x}$, and therefore we can differentiate it using the chain rule.

Assume that $u = \sqrt{x}$. Then $y = \ln u$.

By the chain rule formula,

$dy/dx = dy/du \cdot du/dx$

$dy/dx = d/du (\ln u) \cdot d/dx (\sqrt{x})$

$dy/dx = (1/u) \cdot (1/(2\sqrt{x}))$

$dy/dx = (1/\sqrt{x}) . (1/(2\sqrt{x}))$

dy/dx = 1/(2x) (because u = 1/(2√x)).

y = cos (2x2 + 1).

find the derivative of d/dx (sin 2x), express sin 2x = f(g(x)), where f(x) = sin x and g(x) = 2x

Then by the chain rule formula,

d/dx (sin 2x) = cos 2x · 2 = 2 cos 2x

Quotient Rule

Quotient rule in calculus is a method to find the derivative or differentiation of a function given in the form of a ratio or division of two differentiable functions. That means, we can apply the quotient rule when we have to find the derivative of a function of the form: f(x)/g(x), such that both f(x) and g(x) are differentiable, and g(x) ≠ 0. The quotient rule follows the product rule and the concept of limits of derivation in differentiation directly.

f'(x) = [u(x)/v(x)]' = [v(x) × u'(x) - u(x) × v'(x)]/[v(x)]2

Find f'(x) for the following function f(x) using the quotient rule:

i) f(x) = x2/(x+1).

Here, f(x) = x2/(x + 1)

u(x) = x2

v(x) = (x + 1)

⇒u'(x) = 2x

⇒v'(x) = 1

⇒f'(x) = [v(x)u'(x) - u(x)v'(x)]/[v(x)]2

⇒f'(x) = [(x+1)•2x - x2•1]/(x + 1)2

⇒f'(x) = (2x2 + 2x - x2)/(x + 1)2

⇒f'(x) = (x2 + 2x)/(x + 1)2

ii) f(x)= (x-1)/(x+2)

Applying quotient rule:

f'(x)= (x-1)'(x+2) - (x-1)(x+2)'/ (x+2)^2

f'(x)= (1)(x+2) - (x-1)(x+2)'/ (x+2)^2

f'(x) = (x+2) -(x-1) / (x+2)^2

$= x+2-x+1 / (x+2)^2$

$= 3/(x+2)^2$

Applications of derivatives in daily life

Derivatives are used in various fields like science, engineering, physics, and more. In mathematics, derivatives are used to find the rate of change of one quantity with respect to another. For instance, derivatives can be used to calculate the rate of change of the volume of a cube with respect to its decreasing sides, to determine whether a given function is increasing or decreasing, to find the tangent and normal to a curve, and to calculate the highest and lowest point of the curve in a graph or to know its turning point.

In real life, derivatives are employed in many ways such as:

- to calculate the profit and loss in business using graphs.
- to check the temperature variation.
- to determine the speed or distance covered such as miles per hour, kilometer per hour, etc.
- to estimate the profit or loss of a business.

- Temperature variations as a function of location can be used to forecast weather.

  Derivatives are frequently employed in everyday life to determine the extent to which something is changing. The government employs them in population censuses, many disciplines, and even economics. Knowing how to utilize derivatives, when to use them, and how to use them in everyday life is an essential element of any job.

Conclusion

In conclusion, derivatives, whether in the realm of mathematics or finance, play a pivotal role in our understanding of the world.

In mathematics, the concept of derivatives forms the backbone of calculus, providing us with a tool to understand the instantaneous rate of change of a function. This understanding is crucial in various fields, from physics to engineering, and even in economics.

Therefore, whether we are discussing the slope of a curve at a particular point or the future price of a stock, derivatives help us make sense of change and uncertainty, making them an indispensable part of modern life.