GOLDEN RATIO

PROJECT

Submitted to University of Kerala in partial fulfilment of the requirements for the award of the Degree of Bachelor of Science

In

Mathematics

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CERTIFICATE

This is to certify that the project entitled **"GOLDEN RATIO"** is based on the work carried out by APARNA M, DIYA CHANDANA, SHAJINA MOL, SHINI S M, SUCHITHRA C S under the guidance of Dr. Sebina Mathew, Lecturer in Mathematics, All Saints' College, Thiruvananthapuram and no part of this work has formed the basis for the award of any degree or diploma to any other University.

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- Aparna M
- Diya Chandana
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INTRODUCTION

The golden ratio otherwise known as the Divine Proportion or Phi is a mathematical ratio with special properties and aesthetic significance. An enormous number of things in the universe are engineered around the ratio, ranging from the human body to the ark of covalent to snail shells to the orbits of the planets. The divine ratio and golden rectangles appear throughout the ancient architecture and art. The golden ratio is believed to be the most aesthetically pleasing and harmonious means of design. Statistical analysis indicates that "the people involuntarily give preference to proportions that approximate to the Golden Section (Golden ration)."

The golden ratio was called the extreme and mean ratio by Euclid, and the divine proportion by Luca Pacioli, and also goes by several other names. Mathematicians have studied the golden ratio's properties since antiquity. It is the ratio of a regular pentagon's diagonal to its side, and thus appears in the construction of the dodecahedron and icosahedron. A golden rectangle that is, a rectangle with an aspect ratio of may be cut into a square and a smaller rectangle with the same aspect ratio. The golden ratio has been used to analyze the proportions of natural objects as well as artificial systems such as financial markets, in some cases based on dubious fits to data. The golden ratio appears in some patterns in nature, including the spiral arrangement of leaves and other parts of vegetation.

The Fibonacci numbers are Nature's numbering system. They appear everywhere in Nature, from the leaf arrangement in plants, to the pattern of the florets of a flower, the bracts of a pinecone, or the scales of a pineapple. The Fibonacci numbers are therefore applicable to the growth of every living thing, including a single cell, a grain of wheat, a hive of bees, and even all of mankind. It plays a vital role in the arrangement of petals in flowers, structure of DNA and various proportions in human face, structure of sea shells etc. Occurrence of this proportion in zoology is frequent, viz in the clock cycle of brain waves, in hearing and balance organ etc. Here we wish to explore the mysterious secrets of golden ratio concealed in the human anatomy.

If we take the ratio of two successive numbers in Fibonacci's series, (1, 1, 2, 3, 5, 8, 13 . . .), we will find the following series of numbers: $\frac{1}{1} = 1$, $\frac{2}{1} = 2$, $\frac{3}{2} = 1.5$, $\frac{5}{3} = 1.666\ldots$, $\frac{8}{5} = 1.6$, $\frac{13}{8} = 1.625$, $\frac{21}{13} = 1.61538 \ldots$. It is easier to see what is happening if we plot the ratios on a graph, Refer Figure 1.1.

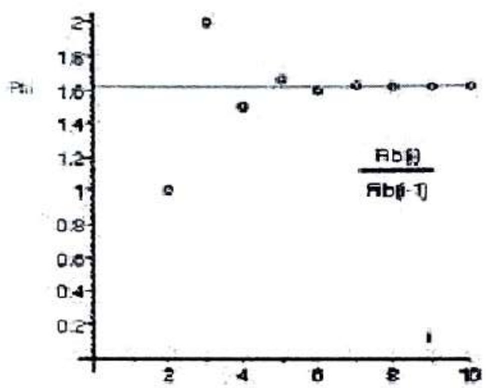

Fig. 1.1: Picturisation of convergence of Fibonacci series.

The ratio seems to be settling down to a particular value, which we call the golden ratio or the golden number. The golden ratio 1.618034… is also called the golden section or the golden mean or just the golden number. It is often represented by a Greek letter Phi φ. The closely related value which we write as phi with a small "p" is just the decimal part of Phi, namely 0.618034…

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HISTORY

Ancient Greek mathematicians first studied the golden ratio because of its frequent appearance in geometry; the division of a line into "extreme and mean ratio" (the golden section) is important in the geometry of regular pentagrams and pentagons. According to one story, 5th-century BC mathematician Hippasus discovered that the golden ratio was neither a whole number nor a fraction (an irrational number), surprising Pythagoreans. Euclid's Elements (c.300 BC) provides several propositions and their proofs employing the golden ratio and contains its first known definition which proceeds as follows:

A straight line is said to have been cut in extreme and mean ratio when, as the whole line is to the greater segment, so is the greater to the lesser.

The golden ratio was studied peripherally over the next millennium. Abu Kamil (c.850–930) employed it in his geometric calculations of pentagons and decagons; his writings influenced that of Fibonacci (Leonardo of Pisa) (c. 1170–1250), who used the ratio in related geometry problems but did not observe that it was connected to the Fibonacci numbers.

Luca Pacioli named his book Divina proportione (1509) after the ratio; the book, largely plagiarized from Piero della Francesca, explored its properties including its appearance in some of the Platonic solids. Leonardo da Vinci, who illustrated Pacioli's book, called the ratio the sectio aurea (golden section). Though it is often said that Pacioli advocated the golden ratio's application to yield pleasing, harmonious proportions, Livio points out that the interpretation has been traced to an error in 1799, and that Pacioli actually advocated the Vitruvian system of rational proportions. Pacioli also saw Catholic religious significance in the ratio, which led to his work's title. 16th-century mathematicians such as Rafael Bombelli solved geometric problems using the ratio. German mathematician Simon Jacob

(d. 1564) noted that consecutive Fibonacci numbers converge to the golden ratio; this was rediscovered by Johannes Kepler in 1608. The first known decimal approximation of the golden ratio was stated as "about" in 1597 by Michael Maestlin of the University of Tübingen in a letter to Kepler, his former student. The same year, Kepler wrote to Maestlin of the Kepler triangle, which combines the golden ratio with the Pythagorean theorem. Kepler said of these:

18th-century mathematicians Abraham de Moivre, Nicolaus I Bernoulli, and Leonhard Euler used a golden ratio-based formula which finds the value of a Fibonacci number based on its placement in the sequence; in 1843, this was rediscovered by Jacques Philippe Marie Binet, for whom it was named "Binet's formula". Martin Ohm first used the German term goldener Schnitt (golden section) to describe the ratio in 1835. James Sully used the equivalent English term in 1875.

By 1910, mathematician Mark Barr began using the Greek letter Phi (φ) as a symbol for the golden ratio. It has also been represented by tau (τ) the first letter of the ancient Greek τομή (cut or section).

3

MATHEMATICS OF GOLDEN RATIO

The golden ratio is a mathematical ratio with special properties and aesthetic significance. In this section the mathematical side of φ is discussed.

Definition 3.1: Two quantities are in the golden ratio if the ratio of the sum of the quantities to the larger quantity is equal to the ratio of the larger quantity to the smaller one.

The figure illustrates the geometric relationship, Refer figure 3.1.

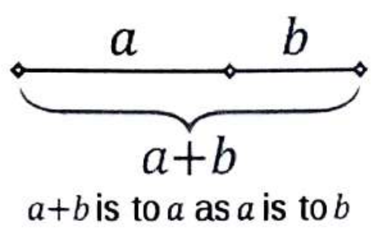

Fig. 3.1: Illustration regarding the Definition 3.1

We can express the figure algebraically as $\frac{a+b}{a} = \frac{a}{b} \equiv \varphi$. Solving this, we obtain $\varphi = 1 + \frac{1}{\varphi}$. That is $\varphi^2 = 1 + \varphi$. Being a quadratic equation in φ, this equation has one positive solution in the set of irrational numbers, $\varphi = \frac{1+\sqrt{5}}{2} \approx 1.6180339887$. At least since the Renaissance many artists and architects have proportioned their works to approximate the golden ratio-especially in the form of the golden rectangle, in which the ratio of the longer side to the shorter is the golden ratio-believing this proportion to be aesthetically pleasing. Mathematicians have studied the golden ratio because of its unique and interesting properties. Construction of a golden rectangle,

(The construction is depicted in the figure 3.2)

1. Construct a unit square (red).

2. Draw a line from the midpoint of one side to an opposite corner.

3. Use that line as the radius to draw an arc that defines the long dimension of the rectangle.

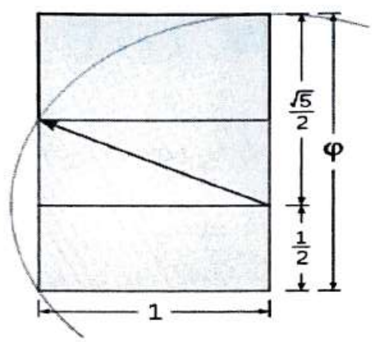

Fig. 3.2: Construction of golden rectangle.

A golden rectangle with longer side a and shorter side b, when placed adjacent to a square with sides of length a, will produce a similar golden rectangle with longer side a+b and shorter side a, See Figure 3.3. This illustrates the relationship: $\frac{a+b}{a} = \frac{a}{b} \equiv \varphi$

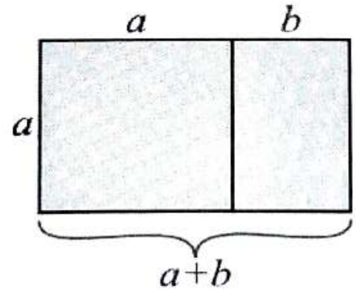

Fig. 3.3: Construction of golden rectangles.

3.1 Various Types of Representations of Golden Ratios

We can represent φ in various forms as given below:

- The formula $\varphi = 1 + \frac{1}{\varphi}$ can be expanded recursively to obtain a continued fraction for the golden ratio:

$$\varphi = [1; 1, 1, 1 \ldots] = 1 + \cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{\cdots\cdots}}}$$

- The equation $\varphi^2 = 1 + \varphi$ likewise produces the continued square root, or infinite surd, form: $\varphi = \sqrt{1+\sqrt{1+\sqrt{1+\cdots}}}$

- An infinite series can be derived to express φ as :

$$\varphi = \frac{13}{8} + \sum_{n=0}^{\infty} \frac{(-1)^{n+1}\,(2n+1)!}{(n+2)!\,n!\;4^{2n+3}}$$

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GOLDEN RATIO IN HUMAN ANATOMY

The Golden Section, also known as Φ (Phi), is manifested throughout the structure of the human body. Some are mentioned below.

4.1 Human Face

There are several golden ratios in the human face.

("ideal human face" determined by scientists and artists).

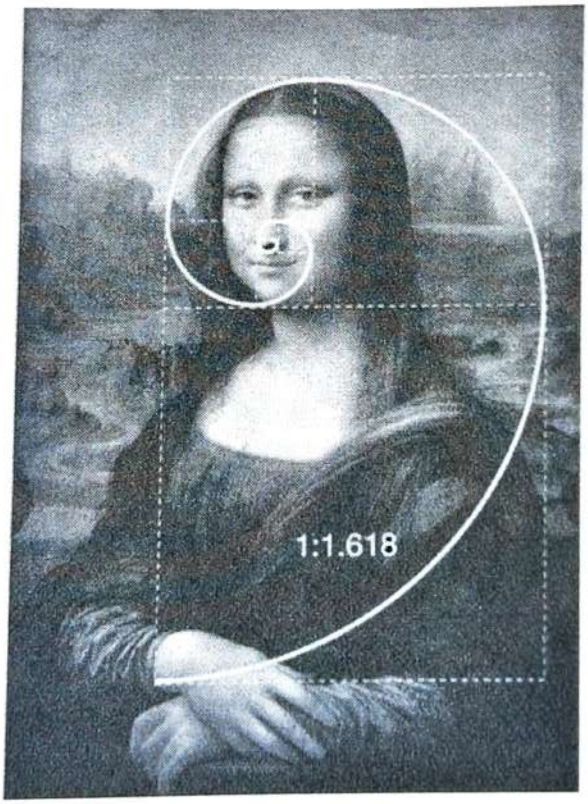

Fig. 4.1: Golden ratio in human face.

For example, the total width of the two front teeth in the upper jaw over their height gives a golden ratio. The width of the first tooth from the center to the second tooth also yields a golden ratio. These are the ideal proportions that a dentist may consider. Some other golden ratios in the human face are:

- Length of face / width of face,
- Distance between the lips and where the eyebrows meet / length of nose,
- Length of face / distance between tip of jaw and where the eyebrows meet,
- Length of mouth / width of nose,
- Width of nose / distance between nostrils,
- Distance between pupils / distance between eyebrows.

4.2 Human Hand

Our fingers have three sections. The proportion of the first two to the full length of the finger gives the golden ratio (with the exception of the thumbs). The proportion of the middle finger to the little finger is also a golden ratio.

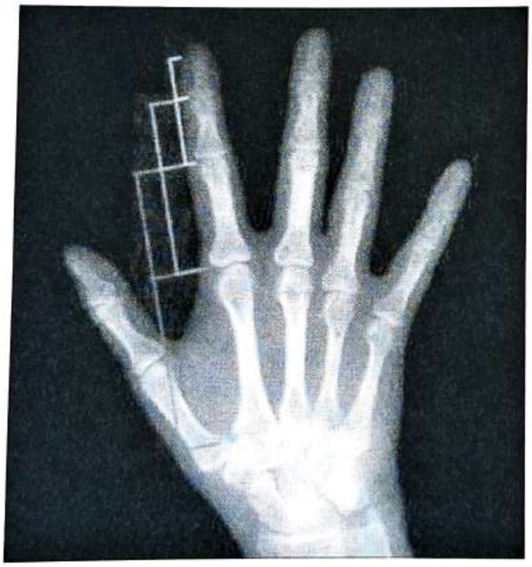

Fig. 4.2: Golden ratio in human hand

We have two hands, and the fingers on them consist of three sections. There are five fingers on each hand, and only eight of these are articulated according to the golden number: 2,3,5 and 8 fit the Fibonacci numbers.

4.3 Lungs

In a study carried out between 1985 1nd 1987, Dr. A L Goldberger, relieved the existence of the golden ratio in the structure of the lung.

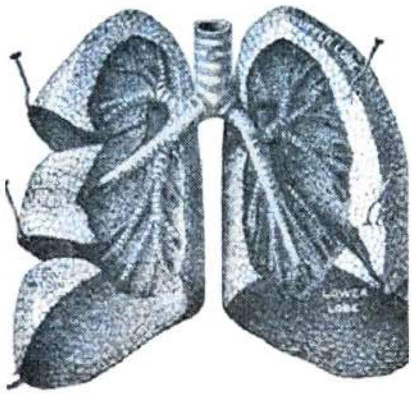

Fig. 4.3: Golden ratio in human lungs.

One feature of the network of the bronchi that constitutes the lung is that it is asymmetric. For example, the windpipe divides into two main bronchi, one long (the left) and other short (the right). This asymmetrical division continues into the subsequent subdivisions of the bronchi. It was determined that in all these divisions the proportion of the short bronchus to the long is always 1/1.618.

4.4 Hearing and Balance Organ

When sound waves enter the ear, they strike the ear drum and cause it to vibrate. Tiny bones in the ear transmit these vibrations to the fluid in the cochlea, where they travel along the narrowing tube those winds into a spiral. The tube is divided into two chambers by an elastic membrane that runs down its length. The mechanical properties of this "basilar" membrane vary from very stiff at the other end and become increasingly flexible as the chambers narrow. These changing properties cause the wave to grow and then die away, much as ocean waves get taller and narrower in shoaling water. Different frequency waves peak at different positions along the tube. Hair cells sitting on the basilar membrane sense these motions by bending against the membrane and produce electrical signals that feed into the auditory nerve.

Hair cells near the large end of the cochlea detect high-pitched sounds, such as the notes of a piccolo, while those at the narrow end of the tube detect lower frequency sounds, like a the oompah of a tuba.

This basic frequency sorting works in the same fashion whether the cochlear tube is laid out straight or coiled in a spiral. That observation, in fact, was the major reason that the researchers studying cochlear mechanics concluded its shape didn't matter. The spiral shape causes the energy in the waves to accumulate against the outside edge of the chamber. D Manoussaki, considered this to be this to the "whispering gallery mode" effect where whispers traveling along curved walls of a large chamber can remain strong enough so they can be heard clearly on the opposite side of the room.

This uneven energy distribution, in turn, causes the fluid to slosh higher on one side of the chamber, forcing the basilar membrane to tilt to one side, the direction to which the hair cells are most sensitive. The effect is strongest in the center of the spiral, where the lowest frequencies are detected.

The researchers calculate the sensitivity increase can be as much as 20 decibels. That corresponds to the difference between the ambience of a quiet restaurant and the noise of a busy street. Spiral-shaped cochleae, are exclusive to mammals. Birds and reptiles generally have plate-like or slightly curved versions of this critical organ, limiting the spans of octaves that they can hear. Animals with tightly coiled cochleae tend to have greater hearing ranges, but previous attempts to associate these auditory effects with the physical characteristics of the cochleae has proven unsatisfactory because they did not take a critical acoustic effect into account.

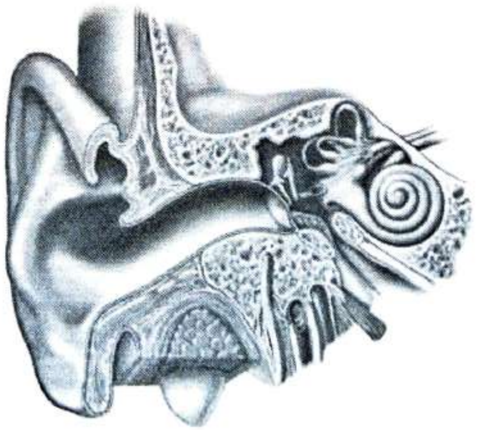

Fig. 4.4: Golden ratio in hearing and balance organ

4.5 Human Genome DNA

The DNA molecule, the program for all life, is based on the golden section. It measures 34 angstroms long by 21 angstroms wide for each full cycle of its double helix spiral. 34 and 21, of course, are numbers in the Fibonacci series and their ratio, 1.6190476 closely approximates phi, 1.6180339. DNA in the cell appears as a double-stranded helix referred to as B-DNA. This form of DNA has a groove in its spirals, with a ratio of phi in the proportion of the major groove to the minor groove, or roughly 21 angstroms to 13 angstroms.

A cross-sectional view from the top of the DNA double helix forms a decagon. A decagon is in essence two pentagons, with one rotated by 36° from the other, so each spiral of the double helix must trace out the shape of a pentagon.

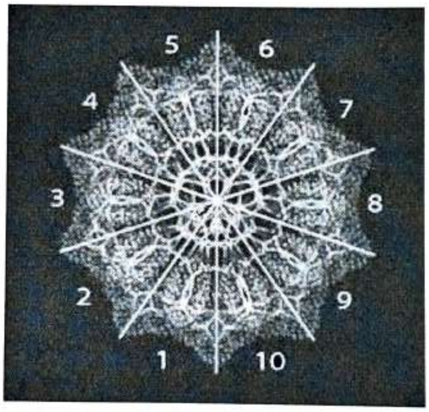

Fig 4.5: A cross-sectional view from the top of the DNA.

Jean-Claude Perez suggests that there is a strong link between DNA and golden ratio in 1991, then again in 1997 in his book. In this work, he shows that the relative proportions of nucleotides within coding DNA sequences like genes or RNA strings are governed by specific Fibonacci and Lucas number sets. This discovery was validated particularly on all known HIV and SIV (Simian Immunodeficiency Virus) retroviruses whole genomes by prof. Luc Montagnier (the discoverer of HIV) which he called the discovery a 'DNA Supracode'.

DNA supracode is revealed by computing sets called "resonances" within any DNA sequence: specific nucleotides clusters like FFF (Fibonacci Fibonacci Fibonacci), LLL (Lucas Lucas Lucas), FFL (Fibonacci Fibonacci Lucas) or LFF (Lucas Fibonacci Fibonacci). An example of elementary FFF resonance: 144 contiguous TCAG bases contain exactly 55 bases T and 89 bases A or C or G. This kind of screening is processed along the DNA sequence, for all possible Fibonacci/Lucas combinations, and for all possible values (example here 144, 89 and 55 are three consecutive Fibonacci numbers). Then there appear lots of "resonances". For example, in HIV whole genome, long of about 9000 bases, there are more than 50000 significant Fibonacci/Lucas resonances. The longer resonances (several hundred resonances are overlapping about 2/3 of the whole HIV genome (6765 bases which is a Fibonacci number). The whole Hyman Genome is controlled by two BINARY CODES ATTRACTORS which provide a kind of self-organized bistable binary code like in computers. The ratio between both bistable states is exactly equal to "2" (the space between two consecutive octaves in music…)

- The top state is exactly matching with a GOLDEN RATIO.
- The Bottom state is also exactly relates to Golden ratio.
- "Top" level = $1/\Phi$
- "Bottom" level = $1/2\Phi$

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CONCLUSION

While some may dispute the significance of the Golden Ratio, it is apparent that through our history there has been a fascination with it. Many will speculate on the validity of it in nature, as well as in our history. It's important to realize that, while although some of these examples mentioned are in fact not quite perfect to Golden Ratio, there still is a significance to the approximate value.

We have come to conclude that it is not the "Holy Grail" of numbers, but merely measurement and we intend to keep using it in our works. If you are looking for an improvement in your own aesthetics, Golden Ratio will be a great source.

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