A STUDY ON THEORY AND FORMATION OF BLACK HOLES

A project report submitted to

KERALA UNIVERSITY

In partial fulfilment of the requirements for the award of the degree of

BACHELOR OF SCIENCE

In

PHYSICS

Submitted by

Reshmi S **Reg. No. 23019100008**

DEPARTMENT OF PHYSICS

ALL SAINTS' COLLEGE

CHACKAI, THIRUVANANTHAPURAM – 695007

DECLARATION

We, the BSc Physics students of **All Saints' College, Thiruvananthapuram** hereby declare that the work presented in this project is an authentic record of our own for the partial fulfilment of the requirement for the award of the BSc Degree in Physics under University of Kerala and has been carried out under the guidance of Dr. Sherin J S, Guest Lecturer, Department of Physics, All Saints' College, Thiruvananthapuram and Dr. Rajina S R, Guest Lecturer, Department of Physics, All Saints' College, Thiruvananthapuram.

Reshmi S **Reg.No. 23019100008**

Reshmi.S

ALL SAINTS' COLLEGE
CHACKAI, THIRUVANANTHAPURAM

CERTIFICATE

Certified that this project entitled **"A STUDY ON THEORY AND FORMATION OF BLACK HOLES"** is a bonafide report of the work done by **Miss Reshmi S** in partial fulfilment of the requirements for the award of the degree, **Bachelor of Science in Physics**, Kerala University during the period 2019-2022.

Head
Dr. Anjana P S
(Associate Professor)
Dept of Physics
All Saints' College
Thiruvananthapuram

Guide
Dr. Sherin J S
(Guest Lecturer)
Dept of Physics
All Saints' College
Thiruvananthapuram

Place: Thiruvananthapuram

Date:

External examiners
1.
2.

ACKNOWLEDGEMENT

It gives me intense pleasure to express my gratitude to all those who encouraged throughout this phase of my project. First of all, I would like to thank the Almighty for blessing and strengthening me to make this project a success.

I would specially thank my guide, Dr. Sherin J S, Guest Lecturer, Department of Physics, All Saints' College, Thiruvananthapuram for her priceless guidance, constant motivation and patience which was immensely beneficial for me during the project tenure.

I express my heartfelt gratitude to Dr. Anjana P S, Associate Professor and Head, Department of Physics, All Saints' College, for her valuable suggestions and constructive criticisms on the project.

I also express my sincere thanks to Dr. Veena Sureshbabu, Assistant Professor, Department of Physics, All Saints' College, for her valuable suggestions on the project.

It is my privilege to express profound gratitude to Dr. Reshmi R Prasad, Principal, All Saints' College Thiruvananthapuram for allowing me to use the technical support along with the lab facilities of the institution.

I would also like to express my sincere gratitude to all Faculty members and non-teaching staff of Department of Physics, All Saints' College for their valuable advice and suggestions.

Last but not the least I express my deepest gratitude to my friends and family, for their love, patience, tolerance, support and encouragement.

CONTENTS

PREFACE

The universe is a vast topic filled with mystery and unexplained facts. The discovery of a theory ends with no solid conclusion.

One of the widely discussed mysteries is the black hole. The simplest definition of a black hole is an object that is so densethat not even light can escape its surface. The concept of a black hole can be understood by thinking about how fast something needs to move to escape the gravity of another object – this is called the escape velocity.

There are two basic parts to a black hole: the singularity and the event horizon. The event horizon is the "point of no return"around the black hole. It is not a physical surface, but a sphere surrounding the black hole that marks where the escape velocity is equal to the speed of light.

There are four types of black holes: stellar, intermediate, supermassive, and miniature. The most commonly known way a black hole forms is by stellar death. As stars reach the ends oftheir lives, most will inflate, lose mass, and then cool to form white dwarfs.

Black holes are objects in the universe with so much mass trapped inside their boundaries that they have incredibly strong gravitational fields. In fact, the gravitational force of a black hole is so strong that nothing can escape once it has gone inside.

CHAPTER 1

INTRODUCTION

The simplest definition of a black hole is an object that is so dense that not even light can escape its surface. Black holes are objects in the universe with so much mass trapped inside their boundaries that they have incredibly strong gravitational fields. In fact, the gravitational force of a black hole is so strong that nothing can escape once it has gone inside. Not even light can escape a black hole, it is trapped inside along with stars, gas, and dust. Most black holes contain many times the mass of our Sun and the heaviest ones can have millions of solar masses.

The concept of a black hole can be understood by thinking about how fast something needs to move to escape the gravity of another object – this is called the escape velocity. Formally, escape velocity is the speed an object must attain to "break free" of the gravitational attraction of another body.

Despite all that mass, the actual singularity that forms the core of the black hole has never been seen or imaged. It is, as the word suggests, a tiny point in space, but it has a LOT of mass. Astronomers are only able to study these objects through their effect on the material that surrounds them. The material around the black hole forms a rotating disk that lies just beyond a region called "the event horizon," which is the gravitational point of no return.

1.1. Structure of a Black Hole

There are two basic parts to a black hole: the singularity and the event horizon. The event horizon is the "point of no return" around the black hole. It is not a physical surface, but a sphere surrounding the black hole that marks where the escape velocity is equal to the speed of light. Its radius is the Schwarzschild radius mentioned earlier.

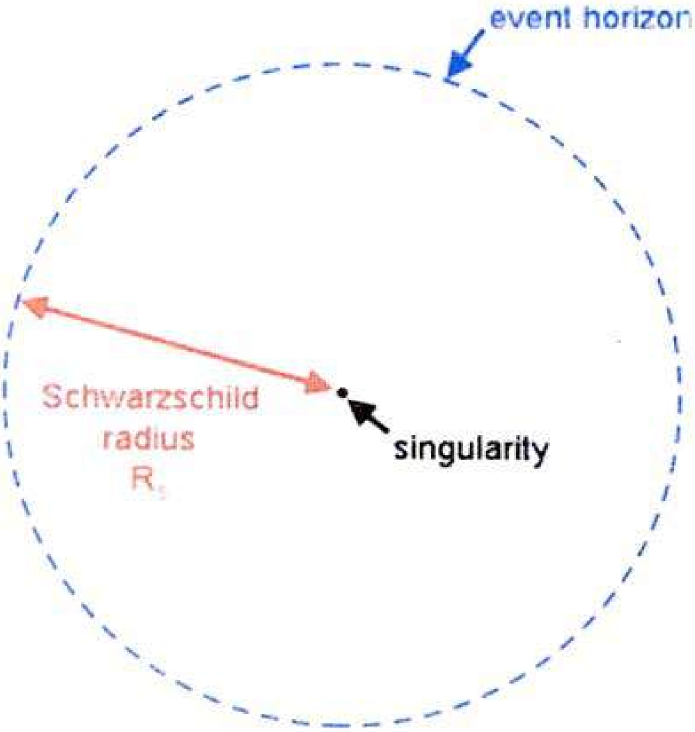

Figure 1.1: Two basic parts of a black hole

The basic "building block" of the black hole is the singularity: a pinpoint region of space that contains all the mass of the black hole. Around it is a region of space from which light cannot escape, giving the "black hole" its name. The outer "edge" of this region is what forms the event horizon. It's the invisible boundary where the pull of the gravitational field is equal to the speed of light. It's also where gravity and speed of light are balanced.

One thing about the event horizon: once matter is inside it, that matter will fall to the center. With such strong gravity, the matter squishes to just a point – a tiny, tiny volume with a crazy-big density. That point is called the singularity. It is vanishingly small, so it has essentially an infinite density. It's likely that the laws of physics break down at the singularity.

The radius at which a mass has an escape velocity equal to the speed of light is called the Schwarzschild radius. The event horizon's position depends on the gravitational pull of the black hole. Schwarzschild radius, also called gravitational radius, the radius below which the gravitational attraction between the particles of a body must cause it to undergo irreversible gravitational collapse. This phenomenon is thought to be the final fate of the more massive stars, leading to formation of black holes.

1.2. THEORY OF BLACK HOLES

Consider stars with very large mass, say 5 to 10 M_{sun}. For such a star, the contraction cannot be arrested either at the white dwarf stage or at the stage of the neutron star. A star may continue to collapse beyond the neutron star stage. When the radius of the star is of the order of 15 kilometer, relativity predicts a most extraordinary phenomenon.

Astronomers calculate the location of an event horizon around a black hole using the equation,

$$R_s = 2GM/c^2$$

where R is the radius of the singularity,

G is the force of gravity,

M is the mass,

c is the speed of light.

Figure 1.2: A supermassive black hole at the core of a galaxy

Figure 1.3 : Simulation of hot gas surrounding and falling into a black hole.

According to the theory of relativity, a ray of light should possess mass and hence be subject to gravity. A ray of light emitted by a star will therefore be pulled back by the star's gravity. If the star is large in size, gravity will not be strong at its surface and a ray of light will be able to escape from the star. If the star shrinks to a size of about15 kilometer radius, the force of gravity at its surface will be billions of times stronger than the force of gravity at the surface of the sun. A ray of light trying to leave the star will therefore be pulled back and it cannot escape from the star. And when light cannot escape, nothing else can escape from it. The star then becomes invisible. It becomes a black hole in space.

The black region in the center of figure 1.2 represents the black hole's event horizon, where no light can escape the massive object's gravitational grip. The black hole's powerful gravity distorts space around it like a funhouse mirror. Light from background stars is stretched and smeared as the stars skim by the black hole.

CHAPTER 2

FORMATION AND ANATOMY OF BLACK HOLES

Traditionally, astronomers have talked about two basic classes of black hole – those with masses about 5-20 times that of the sun, which are called stellar-mass black holes, and those with masses millions to billions times that of the sun, which are called supermassive black holes. There are different types of black holes, and they come about in different ways. The most common type is known as a stellar-mass black hole. These contain roughly up to a few times the mass of our Sun, and form when large main sequence stars (10 - 15 times the mass of our Sun) run out of nuclear fuel in their cores. For a long time astronomers had proposed a third class, called intermediate mass black holes, but it was just in the past decade or so that they have started finding possible evidence of this class of black hole.

Stellar-mass black holes are formed when a massive star runs out of fuel and collapses. They are found scattered throughout the galaxy, in the same places where we find stars, since they began their lives as stars. Some stellar-mass black holes started their lives as part of a binary star system, and the way the black hole affects its companion and their environment can be a clue to astronomers about their presence.

Supermassive black holes are found at the center of nearly every large galaxy. Exactly how supermassive black holes form is an active area of research for astronomers. Recent studies have shown that the size of the black hole is correlated with the size of the galaxy, so that the there must be some connection between the formation of the black hole and the galaxy. The result is a massive supernova explosion that blasts the stars outer layers to space.

2.1. Formation of black holes

Some black holes form when a massive star collapses into itself. A star consists of many layers of gas. At the centre of the star, a burning core exists, where nuclear fusion occurs and joins lighter elements together to form heavier elements. This process generates heat exerting an outward pressure, counteracting the force of gravity that pulls the gastowards the centre of the star. Stars spend most of their lives with these two forces in balance, maintaining its shape and size. You can visualize this process with a hot air balloon that needs the flame to keep the balloon inflated and floating. If the flame goes out, the balloon will collapse and fall from the sky, because there is no more hot air to keep the balloon inflated.

All stars start out fusing hydrogen into helium. Small, cool stars will stop soon after that and will not continue to fuse any other heavier elements. The very hot and more massivestars continue this fusing process to create more massive elements, not only burning hydrogen and helium but also carbon, oxygen and silicon. As the star reaches the end of its life, the nuclear fusion forms iron. Iron is a very stable element and does not easily fuse into heavier elements.

Therefore, it requires much more energy to fuse than it can produce. Therefore, the iron core doesn't fuse into further elements, and the star stops producing energy. When the energy production stops, the force of gravity can finally overcome the outward push from the energy generated by the fusion. As a result, the heavy outer layers of gas of the star are unsupported and the star's core collapses, and in theresulting implosion, blowing apart the rest of the star. The explosion of a star is called supernova. All massive stars will end up this way at the end of their lives, but only the most massive of them will form a black hole.

Black holes can be formed in a number of ways. Some black holes can be formed directly from very big stars, more than twenty-five to hundreds of times bigger than our Sun, when these stars collapse at the end of their lives. After a supernova event, the core of the massive star that is left overafter the explosion is still too massive to support itself againstgravity. Therefore, it continues to collapse causing all of this leftover mass to be compressed into a very small space, forming a black hole. These black holes, typically have mass 3to 100 times the mass of the Sun, and are called stellar-mass black holes. It is thought that there are around 100 million stellar-mass black holes orbiting within our own galaxy.

After their formation, stellar-mass black holes can continue to grow as they accumulate more matter from their surroundings, such as other stars, gas and other black holes.If a black hole absorbs enough material, it can even grow to be more massive than a million Suns. These extremely massive black holes are called "supermassive black holes", and are the largest black holes. Supermassive black holes exist at the centres of most galaxies. One even exists at the centre of the galaxy that we live in, the Milky Way. It's believed that in the conditions of the early Universe, there were a lot of large, short-lived stars, therefore many stellar-mass black holes may have existed, which then gradually accumulated material and merged together over time, creating more massive black holes, eventually containing enough mass to be a supermassive black hole.

While stars come into existence and last for a significantly long time, stars do not exist forever. When stars die matter becomes squeezed into a very tiny space. The effect on such a region is that gravity intensifies to such a degree that no light is able to escape and thus the name black hole came into existence. Hence a black hole is basically formed by the death of a massive star.

2.2. ANATOMY OF A BLACK HOLE

The anatomy of the black hole is a topic worth discussing. The basic parts of a black hole as discussed earlier are the Singularity and the Event horizon. The black holes have other parts too. These are the photon sphere, relativistic jets, innermost stable orbit and the accretion disc. These have been discussed in detail below.

a) SINGULARITY

At the centre of the black hole, matter collapses due to infinite density that occurs due to the strong gravitational pull. This point where the matter becomes squeezed into an infinitely tiny point is called the singularity. All the matter and energy that falls into a black hole ends up here. This prediction of infinite density breaks the laws of physics and there comes the importance of quantum effects, i.e., all the conceptions of space and time collapse.

b) EVENT HORIZON

Event horizon is an imaginary sphere surrounding the singularity where matter and energy cannot escape the black hole's gravity. This is the black portion of the black hole. It has also been defined as the boundary where the velocity needed to escape the black hole's gravitational pull, exceeds the speed of light which is the speed limit of the cosmos. The closer matter is to the black hole greater the speed required to escape from the massive gravity. Hence event horizon is the threshold around the black hole where the escape velocity surpasses the velocity of light.

According to Einstein's special theory of relativity, nothing can exceed the velocity of light. This means that an event horizon is the region from which nothing can return. Its name refers to the impossibility of witnessing any event taking place in that border, the horizon beyond which one cannot see.

c) PHOTON SPHERE

A photon sphere is a region near a black hole where the gravity is extremely strong that even light starts orbiting the black hole. In these unstable orbits, photons can orbit around the black hole a few times, but they will not stay there forever. The event horizon is located at radial coordinate $R_s=2GM/c^2$ (Schwarzschild radius).

Orbiting photons may follow an unstable circular orbit at $3/2R_s$, which is called the photon sphere or a large photon orbit. Any photon orbiting below this distance will plunge into the black hole while light that remains further away will spiral out towards infinity. Depending on the photon's proximity to the last photon orbit it may complete several orbits before spiralling into the event horizon or out towards infinity. As we approach the limit where the photons follow the exact critical orbital radius, the photon will begin orbiting an infinite number of times. According to an observer at infinity, the light from any point would orbit the black hole an arbitrary number of times. This is so because each path of light reaches the observer slightly closer to the edge of the black hole's shadow. Hence the observer would find the entire surface of the event horizon and the entire universe repeating infinitely near the edges of the black hole. This infinite mapping has been studied extensively by scientists using the deflection angle diverging logarithmically in the strong field limit.

d) RELATIVISTIC JETS

When matter is pulled strongly towards the centre of a black hole as it feeds on the surrounding gas and dust, a small fraction of these particles is accelerated to the velocity closer to that of light. This produces powerful jets of radiations that are seen as rushing into the centre of a black hole. In photographs they have been depicted as a flash of light entering into the singularity of a black hole. This normally takes place in a supermassive black hole which is situated in an active galaxy. These jets are believed to be the sources of the fastest moving particles in the universe. Sometimes the jets can surpass through the active galaxy and end in a giant radio lobe that is far away from the centre of this active galaxy. Observed using radio telescopes, they have been found to have various shapes. But mostly they assume a dumb bell shape. These objects are either called radio galaxies or quasars depending on how bright they are and how fast they consume the surrounding matter.

When these black holes grow to become a billion times massive than our Sun, their jets get stronger that they can even expel gas out of a galaxy and stop the formation of new stars. Studies have been conducted on how they are formed and how they sustain. More research is yet to be made about how these jets are capable of producing radiations of such extraordinary broad spectrum up to very high energies.

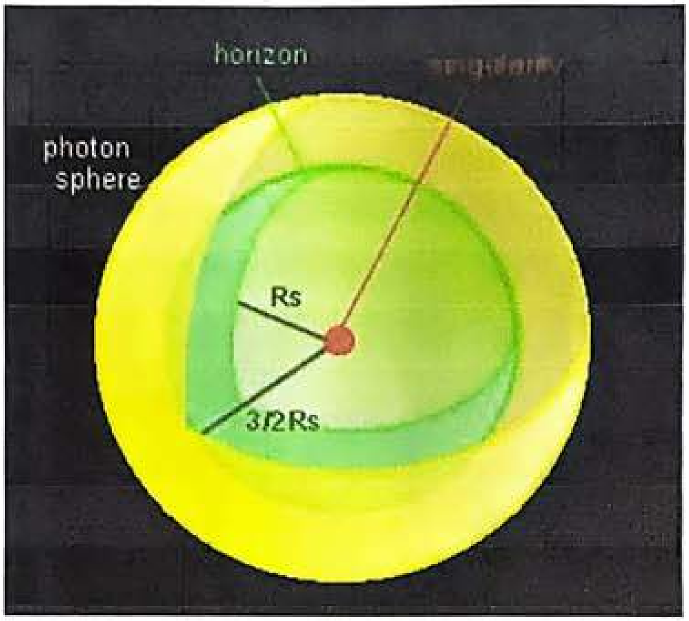

Figure 2.1: Regions of black hole: Singularity, Event horizon and Photon sphere

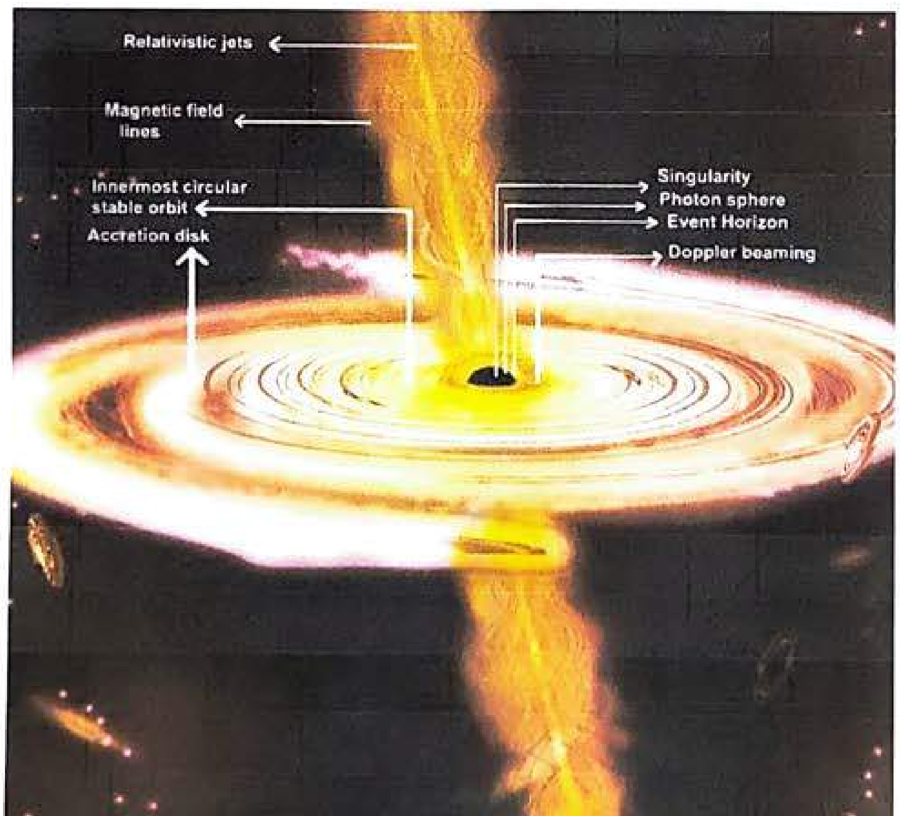

Figure 2.2: Relativistic jets of black hole

e) INNERMOST STABLE CIRCULAR ORBIT (ISCO)

According to Newtonian gravity, all circular orbits are stable. This means that if such an orbit was perturbed slightly, they would only become somewhat elliptical and nothing else is affected. Innermost stable orbits exist in general relativity. For this circular orbit the Schwarzschild radius is 6M, where M is the mass of the Schwarzschild black hole while the ISCO for a Kerr black hole depends on the intrinsic angular momentum a, a well-known Kerr parameter. Inside this radius, a circular orbit is unstable. In the accretion disc (discussed later) theory, the innermost stable circular orbit (ISCO) is discussed as an important feature of a black hole like the singularity, event horizon, etc. ISCO is believed to be the inner edge of an accretion disk orbiting the black hole.

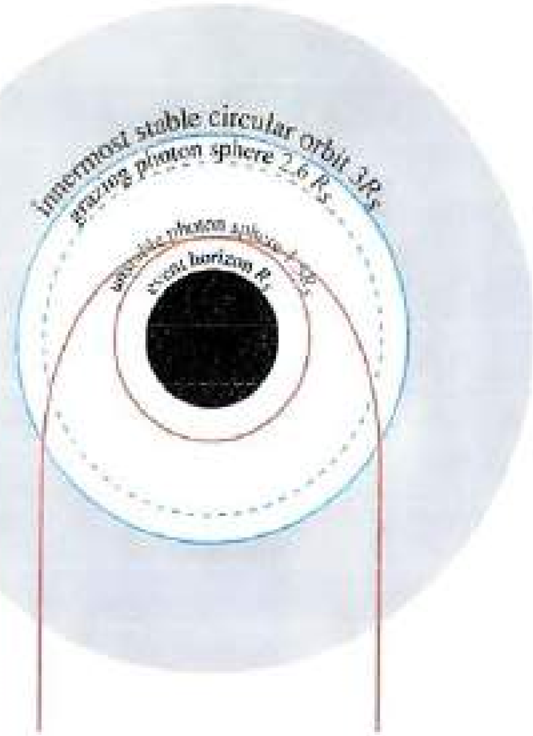

Figure 2.3: ISCO of black hole

f) ACCRETION DISK

A new visualisation of the black hole illustrates how gravity distorts our view. This visualisation gives a picture of a black hole where the in falling matter collects into a thin, hot structure called the accretion disk. Due to the gravitational pull of the black hole, the shape of the disk is distorted. Bright knots are constantly formed and dissipate into the disk as the magnetic fields wind and twist through the churning gas surrounding the black hole. Near a black hole, the gas orbits with the speed close to light. But outside they spin slower than this speed. Due to this difference, it stretches and shears these bright knots producing dark and light lanes in the disk.

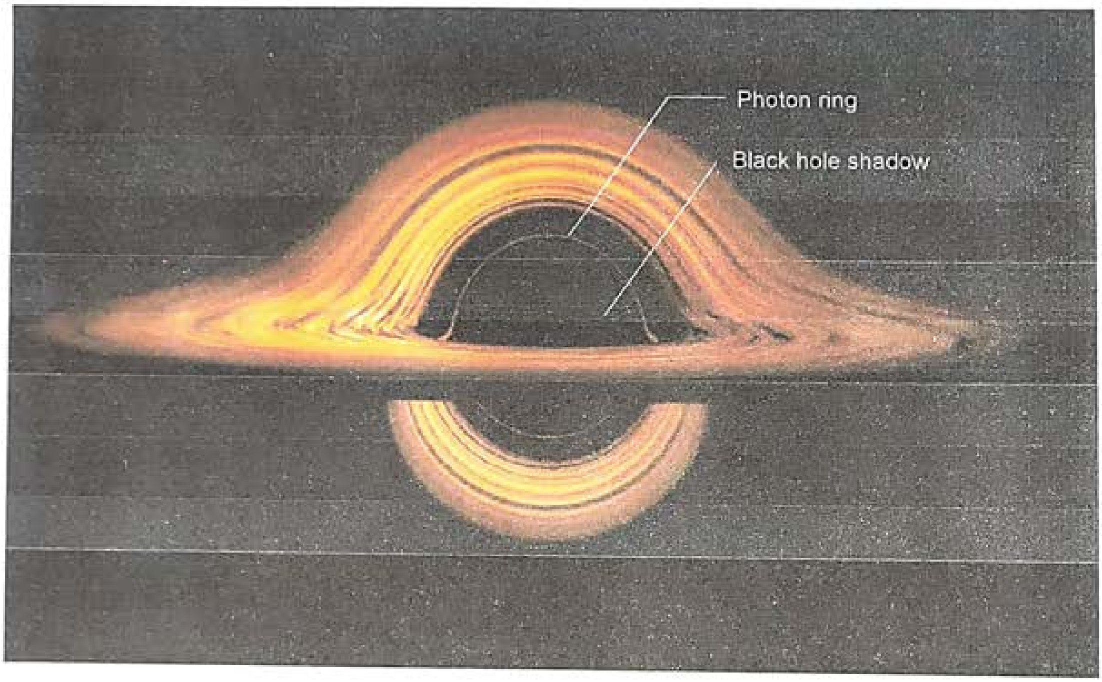

Figure 2.4: Black hole Accretion disk Visualization, NASA

Viewed from the side, the disk looks brighter on the left than on the right. Glowing gas on the left of the disk flows towards us so fast that the effects of Einstein's relativity give a boost in its brightness. On the right, it is the exact opposite. Here the gas is flowing away from us so fast that it appears slightly dimmer than on the left. This asymmetry disappears when we view it exactly face on because none of the material is moving along our line of sight. Closest to the black hole, gravitational light bending becomes so excessive that we can see the underside of the disk as a bright ring of light seemingly outlining the black hole. This so-called "photon ring" is composed of multiple rings that become progressively fainter and thinner, from light that has circled the black hole a large number of times before escaping to reach our eyes. Since the black hole has been pictured as non-rotating and spherical, the photon ring appears circular viewing from any angle. Inside the photon ring is the black hole's shadow, an area which is roughly twice the size of the event horizon.

2.3. TYPES OF BLACK HOLES

There are four types of black holes: Stellar, Intermediate, Supermassive and Miniature. The most commonly known way a black hole forms is by stellar death. As stars reach the ends of their life, mostwill inflate lost mass and then cool to form white dwarfs.

1. Stellar Black holes

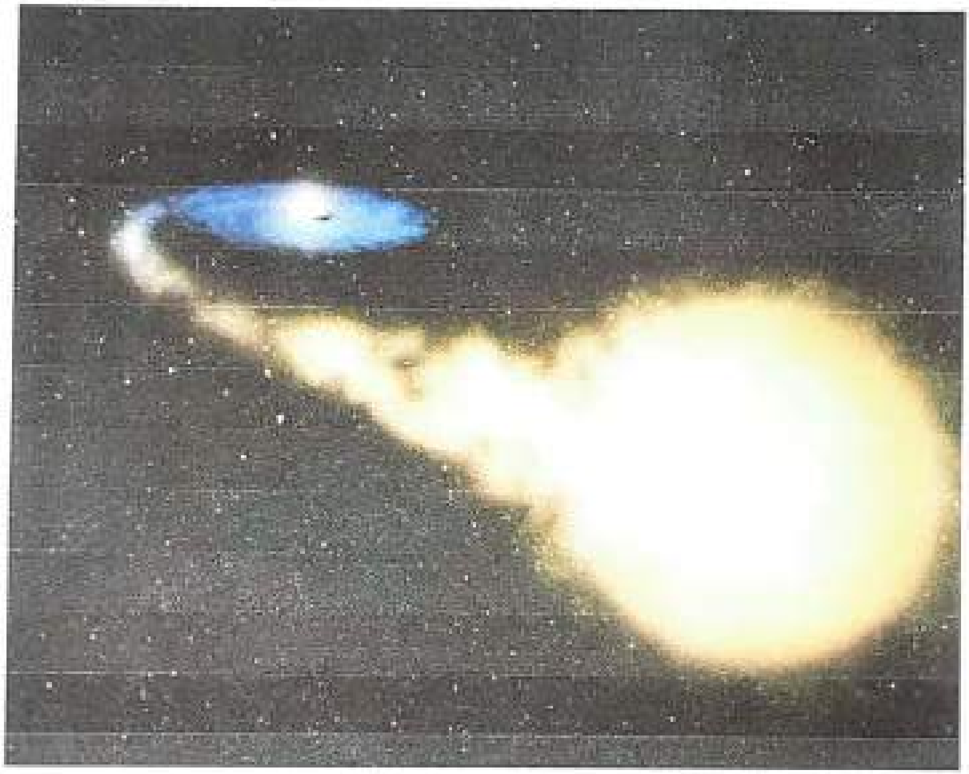

Figure 2.5: A stellar-mass black hole (in blue) hat likely formed when a supermassive star collapsed, feeding from material ejected by a nearby star

When a star burns through the last of its fuel, the object may collapse, or fall into itself. For smaller stars, the new core will become aneutron star or a white dwarf. But when a larger star collapses, it continues to compress and creates a stellar black hole. Black holes formed by the collapse of individual stars are relatively small, but incredibly dense. One of these objects packs more than three times themass of the sun into the diameter of a city. This leads to a crazy amountof gravitational force pulling ion objects around the object. Stellar blackholes then consume the dust and gas from there surrounding galaxies which keep them growing in size.

2. Intermediate Black holes

Figure 2.6: An intermediate-mass black hole

Scientist, once thought that black holes came in only small and large sizes, but recent research revealed the possibility that midsizeor intermediate black holes could exist. Such bodies could form when stars in a cluster collide in a chain reaction. Several of these intermediate black holes forming in the same region could then eventually fall together in the center of a galaxy and create a supermassive black hole.

3. Supermassive Black hole

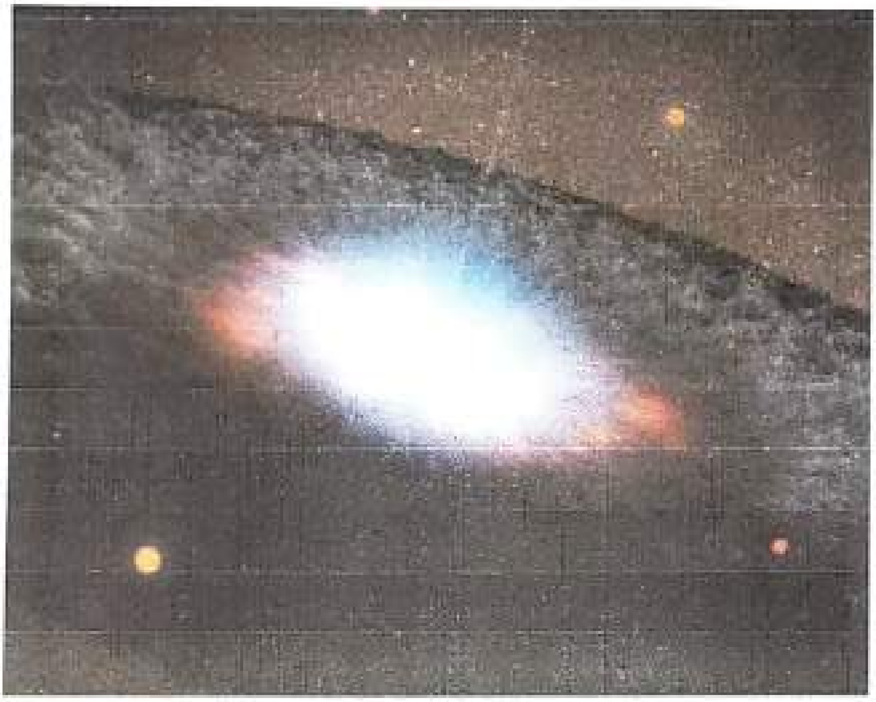

Figure 2.7: A supermassive black holes at the core

Small black holes populate the universe, but their cousins, supermassive black holes, dominate. These enormous black holes are millions or even billions of times as massive as the sun, but are about the same size in diameter. Such black holes are thought to lie at the center of pretty much every galaxy, including the Milky Way. Once these giants have formed, they gather mass from the dust and gas around them, material that is plentiful in the center of galaxies, allowing them to grow to even more enormous sizes. Supermassive black holes may be the result of hundreds or thousands of tiny black holes that merge together. Large gas clouds could also be responsible, collapsing together and rapidly accreting mass. A third option is the collapse of a stellar cluster, a group of stars all falling together. Fourth, supermassive black holes could arise from large clusters of dark matter.This is a substance that we can observe through its gravitational effect on other objects; however, we don't know what dark matter is composed of because it does not emit light and cannot be directly observed.

4. Miniature Black holes

Figure 2.8: Miniature black hole

Miniature black holes are also called quantum mechanical black holes or mini black holes, are hypothetically tiny black holes, for which quantum mechanical effects play an important role. The concept that black holes may exist that are smaller than stellar mass was introduced by Stephan Hawking. Miniature black holes have formed immediately after Big bang. Rapidly expanding space may have squeezed some regions into tiny dense black holes less massive than the sun.

CHAPTER 3

DETECTION AND FACTS ABOUT BLACK HOLES

Light cannot escape from the region around a black hole affected by the event horizon, and also, nobody can really "see" a black hole. However, astronomers can measure and characterize them by the effects they have on their surroundings. Black holes that are near other objects exert a gravitational effect on them. For one thing, mass can also be determined by the orbit of material around the black hole. In practice, astronomers deduce the presence of the black hole by studying how light behaves around it. Black holes, like all massive objects, have enough gravitational pull to bend light's path as it passes by. As stars behind the black hole move relative to it, the light emitted by them will appear distorted, or the stars will appear to move in an unusual way. From this information, the position and mass of the black hole can be determined.

3.1. Detection of black holes

Black holes can be detected by their gravitational influence or by the behavior of matter falling into them. Black holes are assumed to be situated at the center of a galaxy. Our galaxy the Milky way has one such black hole at its center known by the name Sagittarius A*. While observing we can see stars encircling an empty spot. This shows the presence of a black hole due to it gravitational pull.

The second way is by observing the matter falling into the black hole. As matter falls in, it settles in a disk around the black hole that can get very hot. Some of the energy liberated from falling in is turned into light, which we can then see, for example, in X-rays.

Black holes are not so much seen as detected by the affect which they have on the various objects which are around them. This is a method that has been used in science successfully in order to detect extremely small particles not evident to the naked eye. As microscopic and telescopic tools have advanced such approaches have become less important, but in dealing with black holes, areas of no light at all, they have again become extremely useful in black hole research. There are three primary methods currently in effect to detect black holes using their impact on the objects around them. These are mass estimates from objects around a black hole or spiralling in them, gravitational lens effects, and emitted radiation.

Measuring the impact on objects around a black hole can help to identify it and to calculate its size. When objects are near an object of large mass they will often wobble in reaction. When a star or gas disk is seen acting in such a manner when no large mass is nearby this is an indication that a black hole may be present.

Another approach to black hole detection is a gravity lens which draws on Einstein's theory that gravity is actually able to bend space.The outflow from this theory is that while light is not emitted from a black hole, the presence of a black hole has such a significant impact that the light from a gravitational lens would be bent and this could be detected when the lens is focused in the proper location.

Emitted Radiation is yet another method used by scientists to detect black holes. Any material that falls into a black hole is both super-heated as well as accelerated. As a result of this activity, an x-ray telescope will be able to detect the x-rays being emitted from the black hole. In addition to x-rays, emitted material ejected at high speeds can also form jets that can be detected in space.

3.2. FACTS ABOUT BLACK HOLES

a. Black holes are not infinitely small.

b. Black holes are noisy.

c. Black holes might create elements that make life possible.

d. Blacks holes helped create galaxies.

e. You can't see them with the naked eyes.

f. Black holes are only dangerous if you get too close.

g. Dying stars lead to stellar black holes.

h. Black holes don't suck

i. If a star passes too close to a black hole the star can be torn apart.

j. Black holes are ultimate energy factories.

k. There is a supermassive black hole at the center of the galaxy.

l. Black holes slow down time.

m. There are too many black holes to count.

n. Black holes devour things and regularly spit them out.

o. Supermassive black holes give birth to stars.

p. Black holes cannot be destroyed.

q. Black holes remain terrified fodder for science fiction books and movies.

3.3. CONCLUSION

When we look into the universe today, we see that pretty much every large galaxy has a supermassive black hole in its heart. Even the Milky Way has a black hole in it's core with a mass of four million times that of the Sun. Black holes are a long way off at least 26,000 light years. It's mass is still very small compared to the 200 billion solar masses of our galaxy so it can't really harm us. Unless it starts actively feeding, which it isn't. But it might start sometime, if something falls into it. Though we don't anything that can fall into it soon. Even though black holes can cause death and destruction on a major scale they also help galaxies themselves form."

Beyond the event horizon, black holes curve into one of the darkest mysteries in physics. Scientists can't explain what happens when objects cross the event horizon and spiral toward the singularity. General relativity and quantum mechanics collide and Einstein's equations explode into infinities. Black holes might even house gateways to other universes called wormholes and violent fountains of energy and matter called white holes, though it seems very unlikely that nature would allow these structures to exist.

BIBLIOGRAPHY

1. Black Holes on "In Our Time" at the BBC.
2. "A Brief History of Time", Stephen Hawking, Bantam Books,1988
3. "Black Hole Hunters", Overbye Dennis, NASA Archived 2015
4. "Gravity from the ground up", Schutz, Bernard.F, Cambridge University Press,2016
5. "John Michell and black holes", Schaffer and Simon, Journal for the History of Astronomy,2021